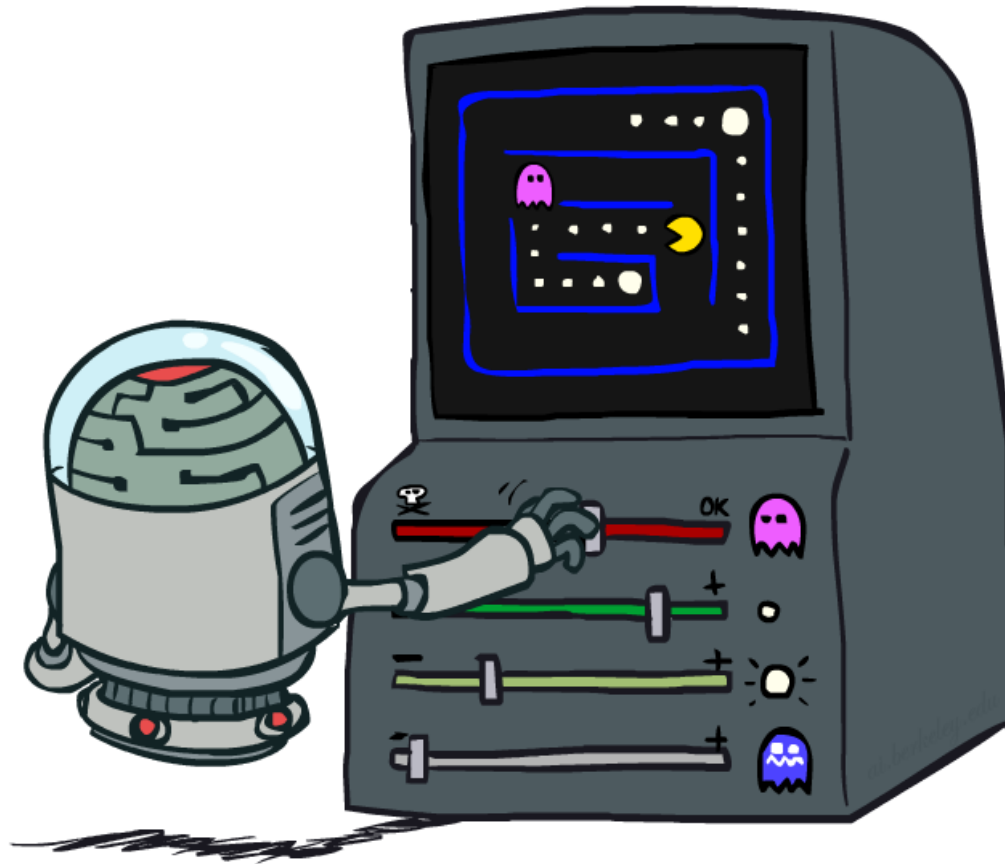


Policy Gradients



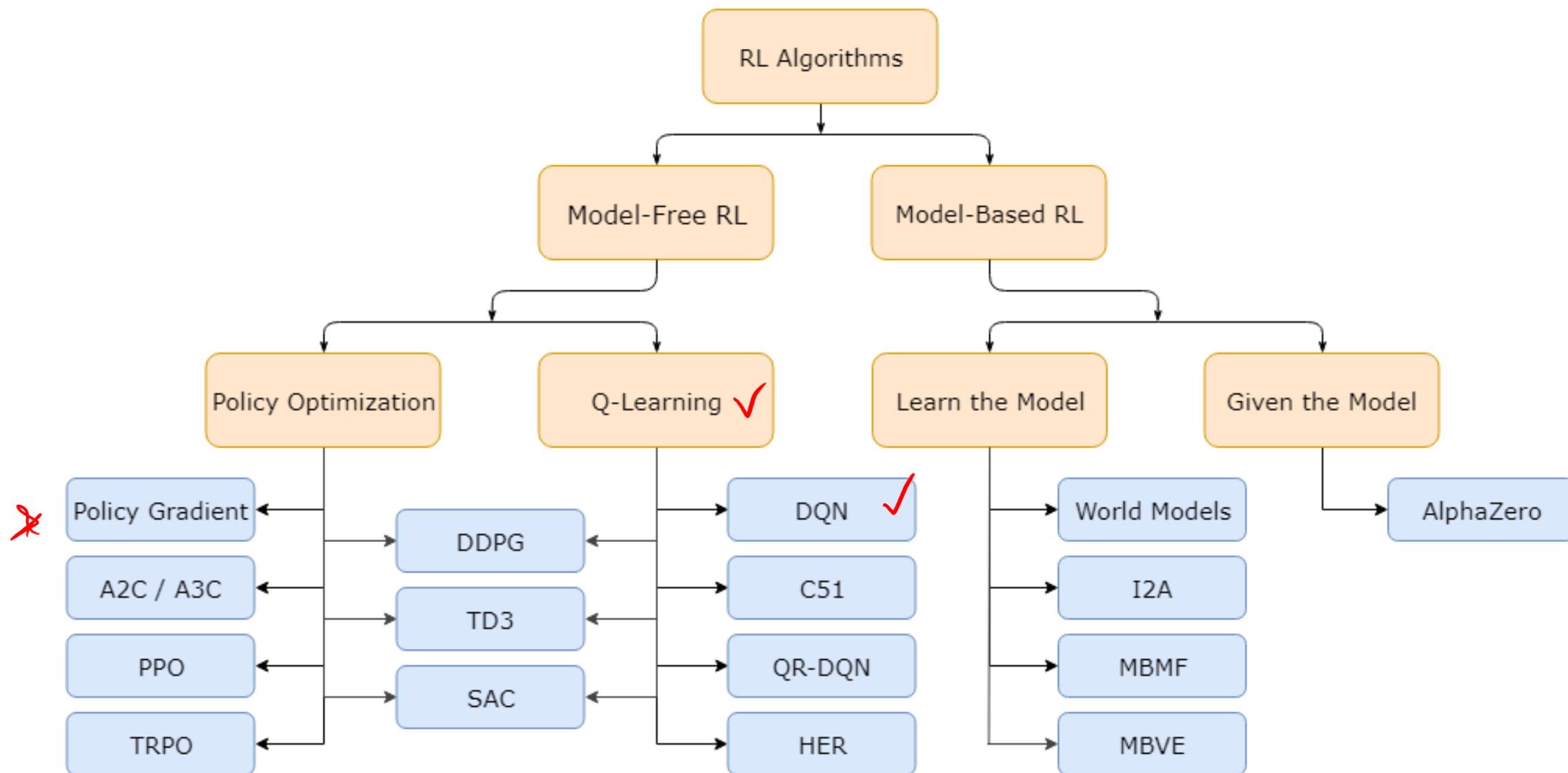
Instructor: Daniel Brown --- University of Utah

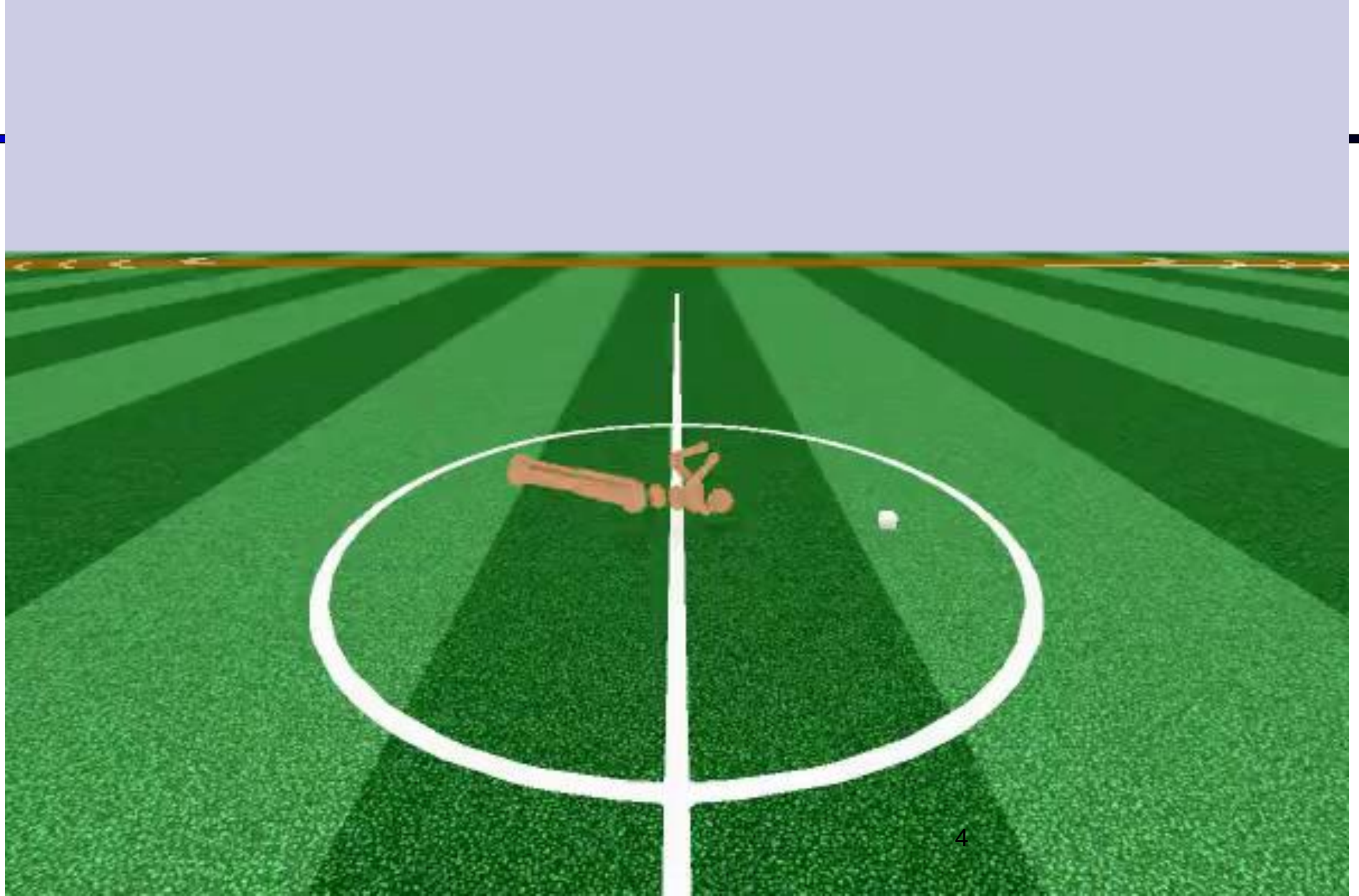
[Based on slides created by Dan Klein and Pieter Abbeel <http://ai.berkeley.edu>.]

Announcement

- Mid semester feedback is open!
- Help me make the class better!
- If you submit feedback you get extra credit equivalent to 2 class quizzes!
 - All feedback is anonymous and appreciated.
 - Confirm submission of feedback via canvas

Rough Taxonomy of RL Algorithms







What is the goal of RL?

- Find a policy that maximizes expected utility (discounted cumulative rewards)

$$\pi^* = \underset{\pi}{\operatorname{argmax}} E_{\pi} \left[\sum_{t=0}^{\infty} \gamma^t R(s, \pi(s), s') \right]$$

Two approaches to model-free RL

■ Learn Q-values

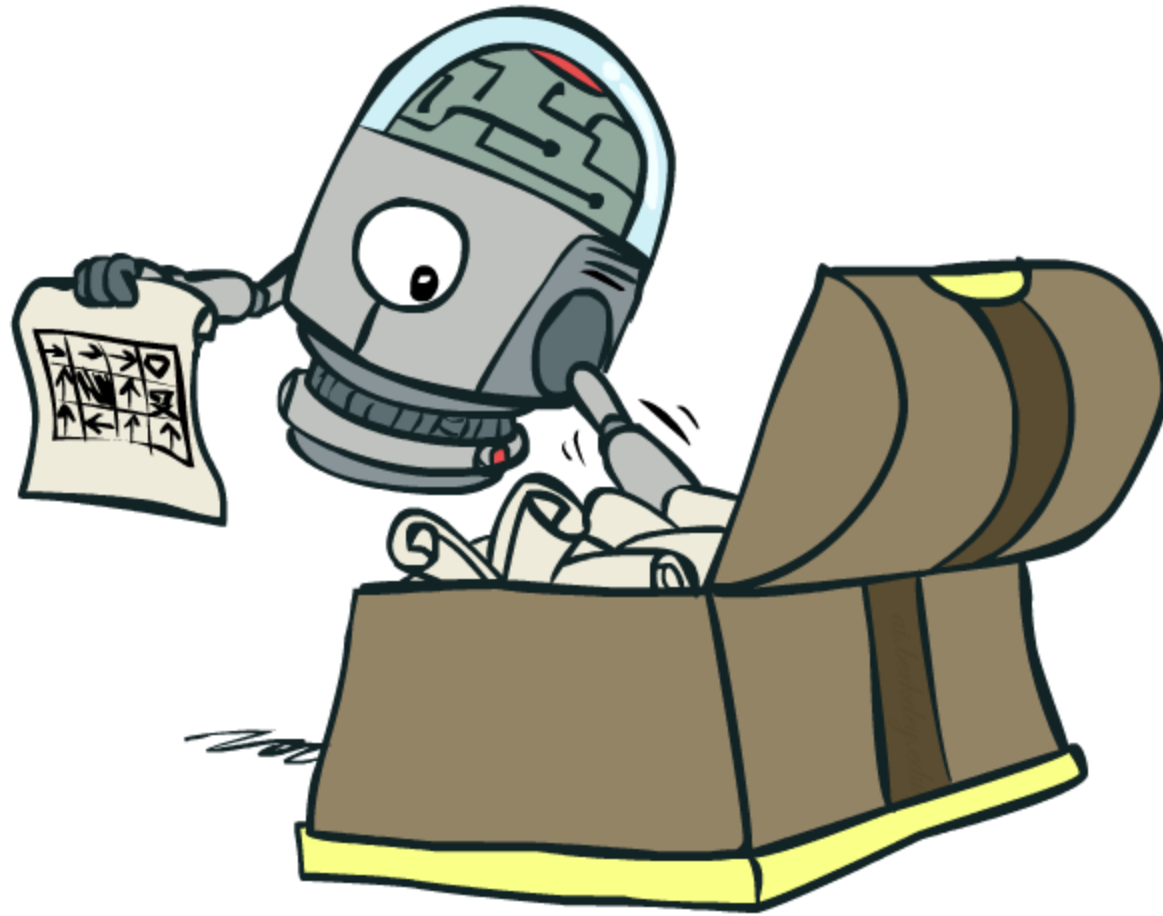
- Trains Q-values to be consistent. Not directly optimizing for performance.
- Use an objective based on the Bellman Equation

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

■ Learn Policy Directly

- Have a parameterized policy π_{θ}
- Update the parameters θ to optimize performance of policy.

Policy Search



Preliminaries

- Trajectory (rollout, episode) $\tau = (s_0, a_0, s_1, a_1, \dots)$

- $s_0 \sim \rho_0(\cdot), \quad s_{t+1} \sim P(\cdot | s_t, a_t)$

- Rewards $r_t = R(s_t, a_t, s_{t+1})$

- Finite-horizon undiscounted return of a trajectory

$$R(\tau) = \sum_{t=0}^T r_t$$

minimize goal

- Actions are sampled from a parameterized policy π_θ

$$a_t \sim \pi_\theta(\cdot | s_t)$$

Preliminaries

Chain Rule of
Prob w/ Markov

- Probability of a trajectory (rollout, episode) $\tau = (s_0, a_0, s_1, a_1, \dots)$

$$P(\tau|\pi) = \overset{\text{initial dist}}{\rho_0(s_0)} \prod_{t=0}^{T-1} P(s_{t+1}|s_t, a_t) \pi_{\theta}(a_t|s_t)$$

- Expected Return of a policy $J(\pi)$

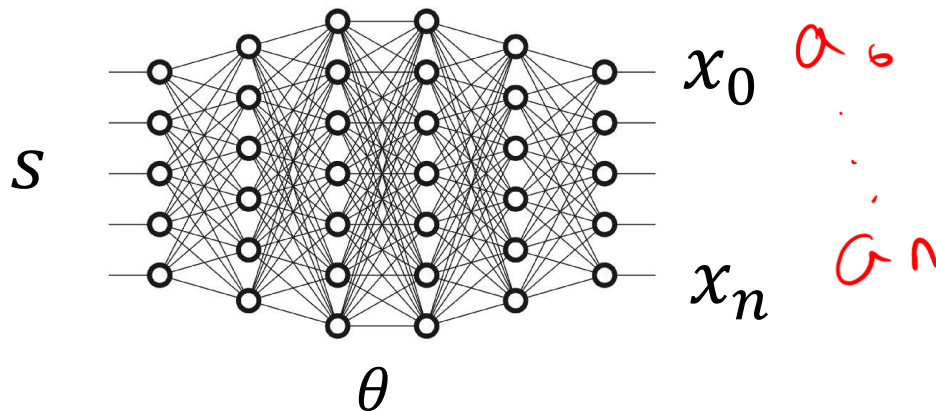
$$J(\pi) = \sum_{\tau} P(\tau|\pi) R(\tau) = E_{\tau \sim \pi}[R(\tau)]$$

- Goal of RL: Solve the following optimization problem

$$\pi^* = \operatorname{argmax}_{\pi} J(\pi)$$

How should we parameterize our policy?

- We need to be able to do two things:
 - Sample actions $a_t \sim \pi_\theta(\cdot | s_t)$
 - Compute log probabilities $\log \pi_\theta(a_t | s_t)$
- Categorical (classifier over discrete actions)
 - Typically, you output a value x_i for each action (class) and then the probability is given by a softmax equation



$$\pi_\theta(a_i | s) = \frac{\exp(x_i)}{\sum_j \exp(x_j)}$$

How should we parameterize our policy?

- Diagonal Gaussian (distribution over continuous actions)

$$a \sim N(\mu, \Sigma)$$

where Σ has non-zero elements only on the diagonal.

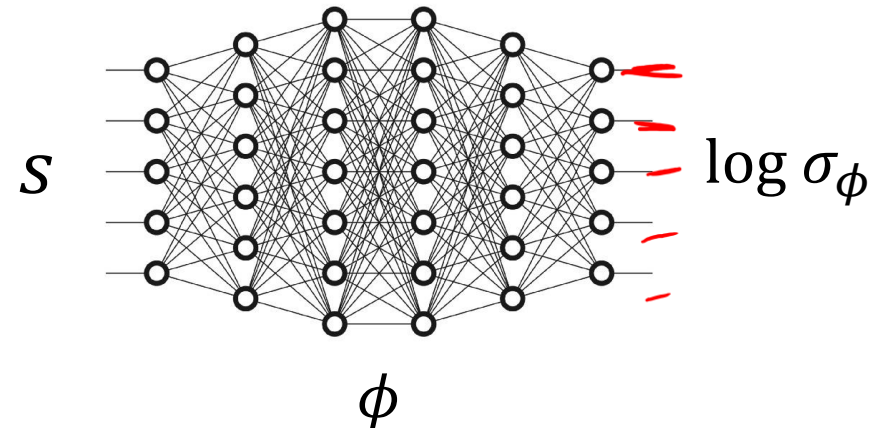
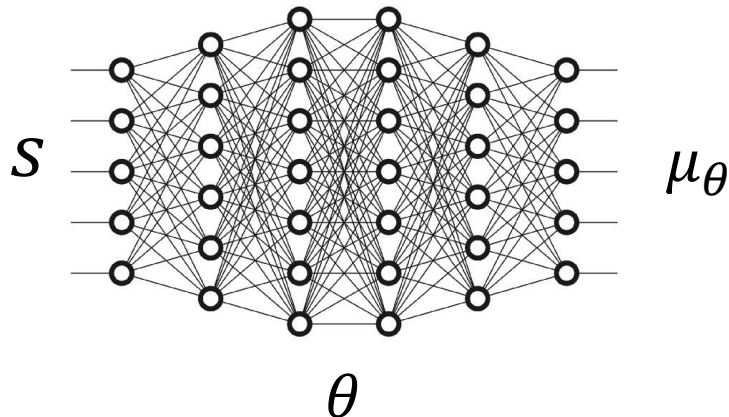
Thus, an action can be sampled as

$$a = \mu_{\theta}(s) + \sigma_{\phi}(s) \odot z,$$

element wise
mult

$$z \sim N(0, I)$$

$$\sigma_{\phi} = \exp(\log \sigma_{\phi})$$



Goal: Update Policy via Gradient Ascent

- We have a parameterized policy and we want to update it so that it maximizes the expected return.
- We want to find the gradient of the return with respect to the policy parameters and step in that direction.

$$\theta_{k+1} \leftarrow \theta_k + \alpha \nabla_{\theta} J(\pi_{\theta}) \Big|_{\theta_k}$$

Policy gradient

Fact #1

- Probability of a trajectory:

- The probability of a trajectory $\tau = (s_0, a_0, \dots, s_{T+1})$ given that actions come from π_θ is

$$P(\tau|\pi) = \rho_0(s_0) \prod_{t=0}^{T-1} P(s_{t+1}|s_t, a_t) \pi_\theta(a_t|s_t)$$

Fact #2

$$\log(A \cdot B) = \log A + \log B$$

- Log-probability of a trajectory:

- The log-probability of a trajectory $\tau = (s_0, a_0, \dots, s_{T+1})$ given that actions come from π_θ is

$$\begin{aligned} \log P(\tau|\pi) &= \log \left(\rho_0(s_0) \prod_{t=0}^T P(s_{t+1}|s_t, a_t) \pi_\theta(a_t|s_t) \right) \\ &= \log \rho_0(s_0) \\ &\quad + \sum_{t=0}^T (\log P(s_{t+1}|s_t, a_t) + \log \pi_\theta(a_t|s_t)) \end{aligned}$$

Fact #3

- Grad-Log-Prob of a Trajectory

- Note that gradients of everything that doesn't depend on θ is 0.

$$\nabla_{\theta} \log P(\tau|\theta) = \nabla_{\theta} \log \rho_0(s_0) + \sum_{t=0}^T (\nabla_{\theta} \log P(s_{t+1}|s_t, a_t)) + \nabla_{\theta} \log \pi_{\theta}(a_t|s_t))$$

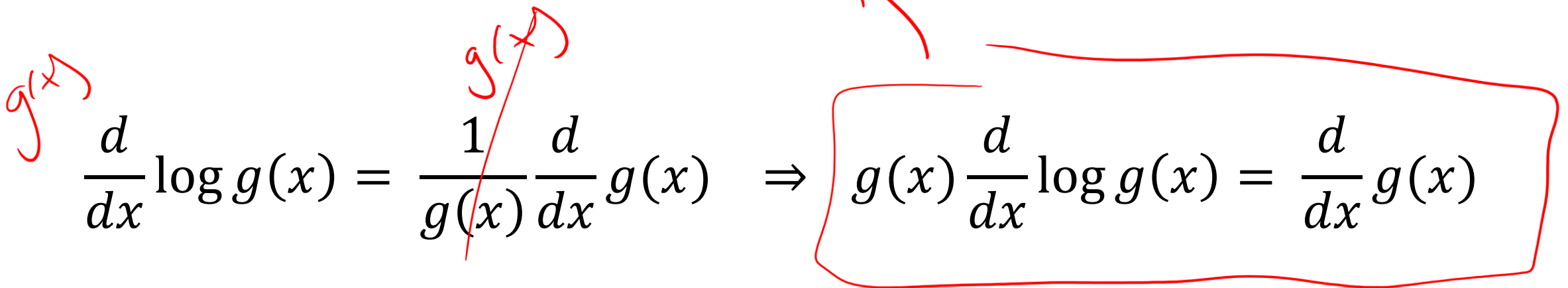
$$= \sum_{t=0}^T (\nabla_{\theta} \log \pi_{\theta}(a_t|s_t))$$

Fact #4

- Log-Derivative Trick:

- This is based on the rule from calculus that the derivative of $\log x$ is $1/x$

$$\nabla_{\theta} P(\tau|\pi) = P(\tau|\pi) \nabla_{\theta} \log P(\tau|\theta)$$



Handwritten red annotations for the log-derivative trick derivation:

- A red $g(x)$ is written above the first $\frac{d}{dx}$ in the first equation.
- A red $g(x)$ is written above the $g(x)$ in the denominator of the first equation, with a red line striking through it.
- A red arrow points from the boxed second equation up to the $\nabla_{\theta} \log P(\tau|\theta)$ term in the main equation above.

$$\frac{d}{dx} \log g(x) = \frac{1}{g(x)} \frac{d}{dx} g(x) \Rightarrow g(x) \frac{d}{dx} \log g(x) = \frac{d}{dx} g(x)$$

$\pi_\theta \triangleq \theta$

Derivation of Policy Gradient

$$\begin{aligned}\nabla_\theta J(\pi_\theta) &= \nabla_\theta E_{\tau \sim \pi_\theta} [R(\tau)] \\ &= \nabla_\theta \sum_\tau P(\tau|\theta) R(\tau) \\ &= \sum_\tau \nabla_\theta P(\tau|\theta) R(\tau) \\ &= \sum_\tau P(\tau|\theta) \nabla_\theta \log P(\tau|\theta) R(\tau) \\ &= E_{\tau \sim \pi_\theta} [\nabla_\theta \log P(\tau|\theta) R(\tau)] \\ &= E_{\tau \sim \pi_\theta} [\sum_{t=0}^T \nabla_\theta \log \pi_\theta(a_t|s_t) R(\tau)]\end{aligned}$$

Handwritten notes:

- $\sum_{t=0}^T r_t$ (above the first line)
- def Expectation (pointing to the first line)
- Fact #4 (pointing to the transition from the 4th to 5th line)
- Fact #3 (pointing to the final line)

The Policy Gradient (REINFORCE)

- We can now perform gradient ascent to improve our policy!

$$\theta_{k+1} \leftarrow \theta_k + \alpha \nabla_{\theta} J(\pi_{\theta}) \Big|_{\theta_k}$$

$$\nabla_{\theta} J(\pi_{\theta}) = E_{\tau \sim \pi_{\theta}} \left[\sum_{t=0}^T \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) R(\tau) \right]$$

Estimate with a
sample mean over a
set D of policy rollouts
given current
parameters

$$\approx \frac{1}{|D|} \sum_{\tau \in D} \sum_{t=0}^T \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) R(\tau)$$

How would you implement this?

1. Start with random policy parameters θ_0
2. Run the policy in the environment to collect N rollouts (episodes) of length T and save returns of each trajectory.

$$a_t \sim \pi_{\theta}(\cdot | s_t) \Rightarrow (s_0, a_0, r_0, s_1, a_1, r_1, \dots, r_T, s_{T+1})$$

$$D = \{\tau_1, \dots, \tau_N\}, \quad R = \{R(\tau_1), \dots, R(\tau_N)\}$$

3. Compute policy gradient

$$\nabla_{\theta} J(\pi_{\theta}) = E_{\tau \sim \pi_{\theta}} \left[\sum_{t=0}^T \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) R(\tau) \right]$$

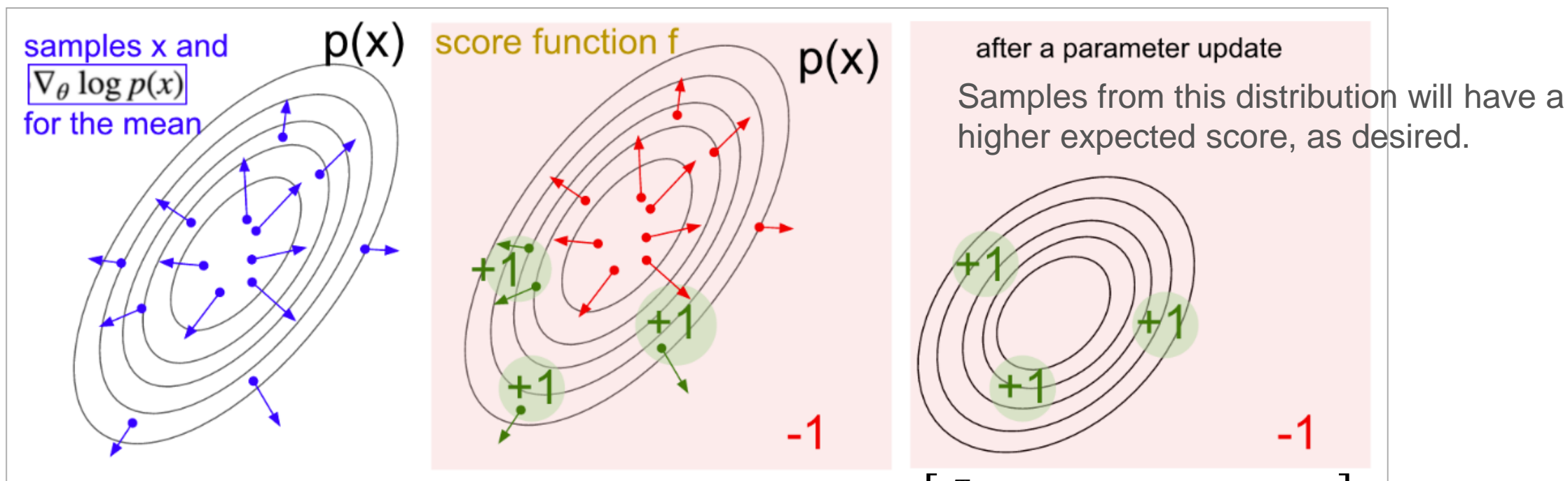
4. Update policy parameters

$$\theta_{k+1} \leftarrow \theta_k + \alpha \nabla_{\theta} J(\pi_{\theta}) \Big|_{\theta_k}$$

5. Repeat from step 2

Some more intuition (thanks to Andrej Karpathy)

- Blue Dots: samples from Gaussian
- Blue arrows: gradients of the log probability with respect to the gaussian's mean parameter
- We score each sample
- Red have score -1
- Green have scores +1
- To update the Gaussian mean parameter, we average up all the green arrows, and the *negative* of the red arrows.



$$\nabla_{\theta} J(\pi_{\theta}) = E_{\tau \sim \pi_{\theta}} \left[\sum_{t=0}^T \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) R(\tau) \right]$$

Policy Gradient RL Algorithms

- We can directly update the policy to achieve high reward.
- Pros:
 - Directly optimize what we care about: Utility!
 - Naturally handles continuous action spaces!
 - Can learn specific probabilities for taking actions.
 - Often more stable than value-based methods (e.g. DQN).
- Cons:
 - On-Policy -> Sample-inefficient we need to collect a large set of new trajectories every time the policy parameters change.
 - Q-Learning methods are usually more data efficient since they can reuse data from any policy (Off-Policy) and can update per sample.

Many forms of policy gradients

$(s_0, a_0, r_0, s_1, a_1, r_1, s_2)$

$$\nabla_{\theta} J(\pi_{\theta}) = \mathbb{E}_{\tau \sim \pi_{\theta}} \left[\sum_{t=0}^T \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \Phi_t \right]$$

weight

What we derived: $\Phi_t = R(\tau)$,

Follows a similar derivation:

$$\Phi_t = \sum_{t'=t}^T R(s_{t'}, a_{t'}, s_{t'+1}),$$

<https://medium.com/@thechrisyoon/deriving-policy-gradients-and-implementing-reinforce-f887949bd63>

- What is better about the second approach? *Reward to go*

- Focuses on rewards in the future!
- Less variance -> less noisy gradients.

Many forms of policy gradients

$$\nabla_{\theta} J(\pi_{\theta}) = \mathbb{E}_{\tau \sim \pi_{\theta}} \left[\sum_{t=0}^T \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \Phi_t \right]$$

$$\Phi_t = \sum_{t'=t}^T R(s_{t'}, a_{t'}, s_{t'+1}),$$

Looks familiar....

$$\Phi_t = Q^{\pi_{\theta}}(s_t, a_t)$$

- Now we have an approach that combines a parameterized policy and a parameterized value function!

I rotate
the piece



Really bad
action



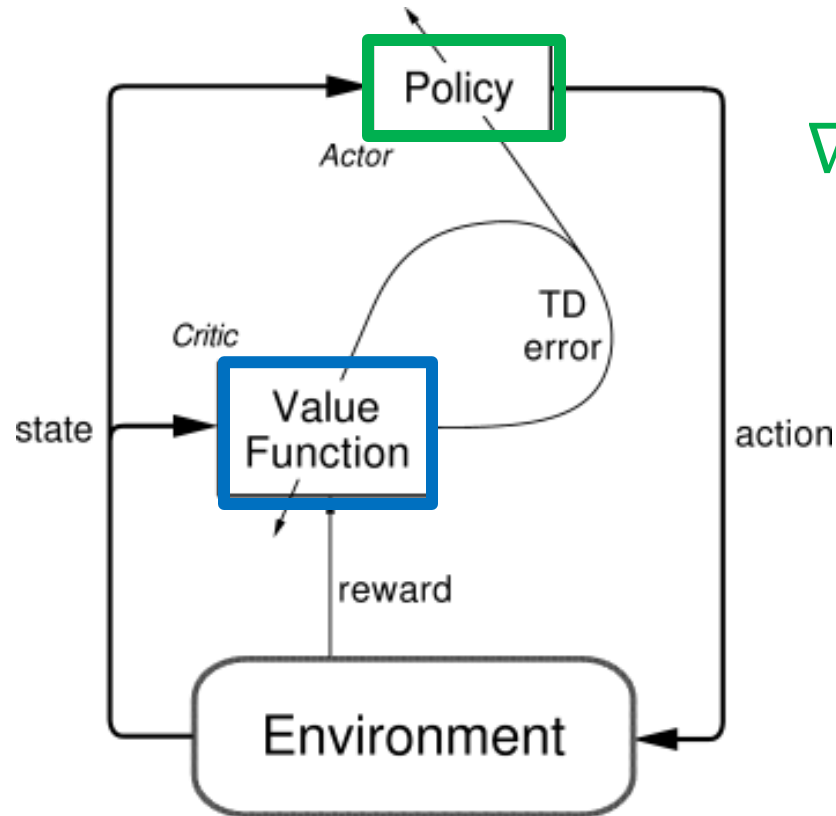
Actor



Critic

Actor Critic Algorithms

- Combining value learning with direct policy learning
 - One example is policy gradient using the advantage function



$$\nabla_{\theta} J(\pi_{\theta}) = E_{\tau \sim \pi_{\theta}} \left[\sum_{t=0}^T \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) Q_w^{\pi_{\theta}}(s_t, a_t) \right]$$

$$\theta_{k+1} \leftarrow \theta_k + \alpha \nabla_{\theta} J(\pi_{\theta}) \Big|_{\theta_k}$$

$$\delta = (r_t + \gamma Q_w^{\pi_{\theta}}(s_{t+1}, a_{t+1}) - Q_w^{\pi_{\theta}}(s_t, a_t))$$

$$w_{k+1} \leftarrow w_k + \alpha \delta_t \nabla_{\theta} Q_w^{\pi_{\theta}}$$

Q Actor Critic Algorithm Pseudo Code

Algorithm 1 Q Actor Critic

Initialize parameters s, θ, w and learning rates α_θ, α_w ; sample $a \sim \pi_\theta(a|s)$.

for $t = 1 \dots T$: **do**

 Sample reward $r_t \sim R(s, a)$ and next state $s' \sim P(s'|s, a)$

 Then sample the next action $a' \sim \pi_\theta(a'|s')$

 Update the policy parameters: $\theta \leftarrow \theta + \alpha_\theta Q_w(s, a) \nabla_\theta \log \pi_\theta(a|s)$; Compute the correction (TD error) for action-value at time t:

$$\delta_t = r_t + \gamma Q_w(s', a') - Q_w(s, a)$$

 and use it to update the parameters of Q function:

$$w \leftarrow w + \alpha_w \delta_t \nabla_w Q_w(s, a)$$

 Move to $a \leftarrow a'$ and $s \leftarrow s'$

end for

Adapted from Lilian Weng's post "Policy Gradient algorithms"

Many forms of policy gradients

$$\nabla_{\theta} J(\pi_{\theta}) = \mathbb{E}_{\tau \sim \pi_{\theta}} \left[\sum_{t=0}^T \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \Phi_t \right]$$

$$\Phi_t = R(\tau), \quad \Phi_t = \sum_{t'=t}^T R(s_{t'}, a_{t'}, s_{t'+1}), \quad \Phi_t = Q^{\pi_{\theta}}(s_t, a_t)$$

$$\Phi_t = \sum_{t'=t}^T R(s_{t'}, a_{t'}, s_{t'+1}) - b(s_t)$$

$$\Phi_t = A^{\pi}(s_t, a_t) = Q^{\pi}(s_t, a_t) - V^{\pi}(s_t)$$

Advantage Function

The Advantage Function

$$A(s, a) = \underbrace{Q(s, a)}_{\text{q value for action a in state s}} - \underbrace{V(s)}_{\text{average value of that state}}$$

- Why good?
- Why bad?

The Advantage Function

$$A(s, a) = \boxed{Q(s, a)} - V(s)$$

$$\frac{r + \gamma V(s')}{\quad}$$

$$A(s, a) = \underline{r + \gamma V(s') - V(s)}$$

TD Error

Advantage Actor Critic (A2C)

- Combining value learning with direct policy learning
 - One example is policy gradient using the advantage function

$$\nabla_{\theta} J(\pi_{\theta}) = \mathbb{E}_{\tau \sim \pi_{\theta}} \left[\sum_{t=0}^T \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \Phi_t \right]$$

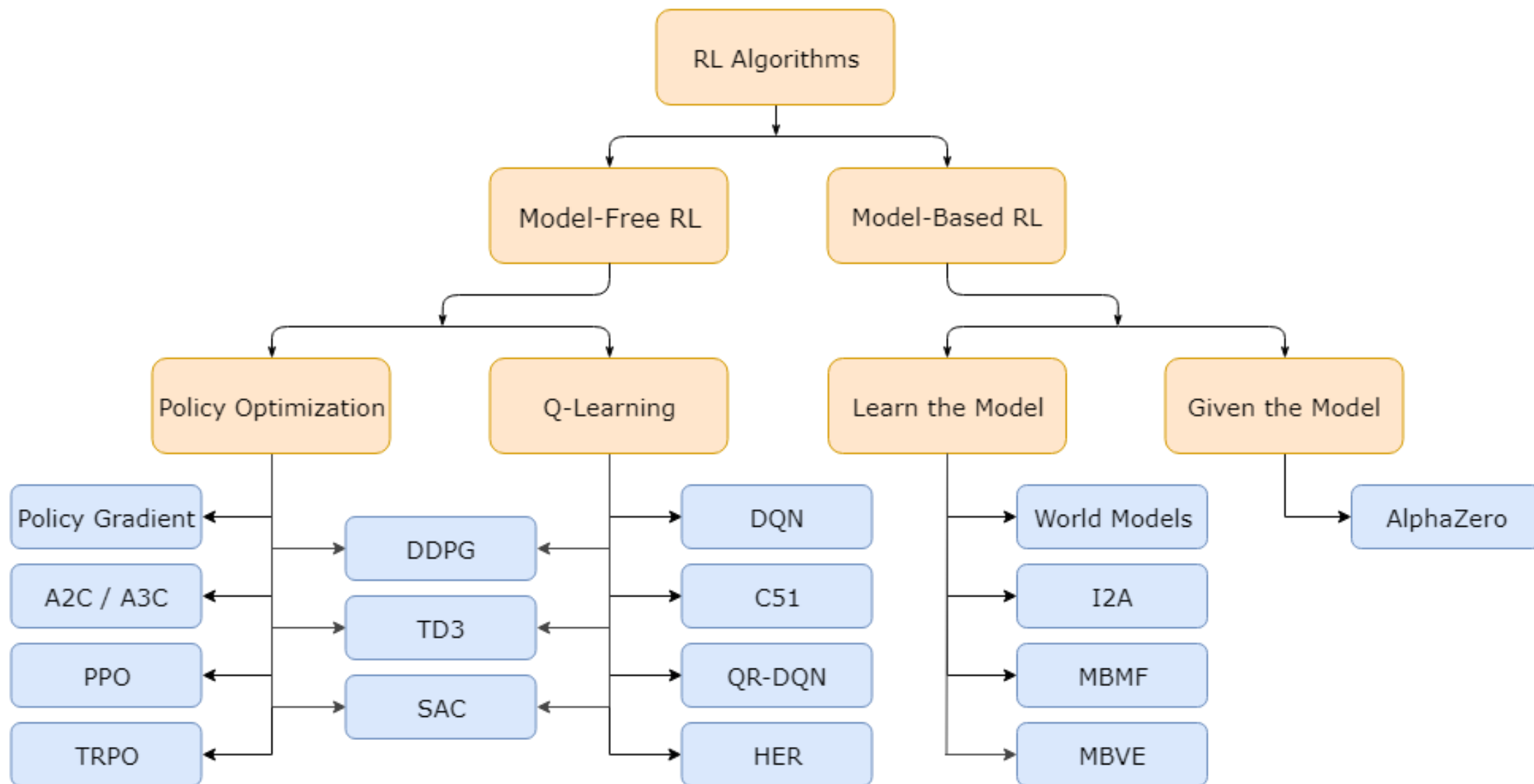
$$\Phi_t = A^{\pi}(s_t, a_t) = Q^{\pi}(s_t, a_t) - V^{\pi}(s_t)$$

$$\text{TD error } \delta_t = r(s_t, a_t) + \gamma V^{\pi}(s_{t+1}) - V^{\pi}(s_t)$$

TD-Learning update

$$w_{k+1} \leftarrow w_k + \alpha \delta_t \nabla_w V(s, a; w)$$

Rough Taxonomy of RL Algorithms



-
- Next time: Alpha Go