## Inverse RL and Reward Learning



#### Instructor: Daniel Brown

[Some slides adapted from Sergey Levine (CS 285) and Alina Vereshchaka (CSE4/510)]

### Reward Learning (Inverse Reinforcement Learning)



#### Why not just imitate behavior? (Behavioral Cloning)





#### Human Intent Inference



#### Inverse Reinforcement Learning

- Given
  - MDP without a reward function
  - Demonstrations from an optimal policy  $\pi^*$
- Recover the reward function *R* that makes  $\pi^*$  optimal

## Imitation Learning

#### **Behavioral Cloning**



$$\Rightarrow \pi$$

- Answers the "How?" question
- Mimic the demonstrator
- Learn mapping from states to actions
- Computationally efficient
- Compounding errors

#### Inverse Reinforcement Learning



 $\Rightarrow R \Rightarrow \pi$ 

- Answers the "Why?" question
- Explain the demonstrator's behavior
- Learn a reward function capturing the demonstrator's intent
- Can require lots of data and compute
- Better generalization. Can recover from arbitrary states

# IRL Example: Teaching a robot to navigate through demonstrations













## Toy version





























#### Inverse Reinforcement Learning Formalism

- Given
  - MDP without a reward function
  - Demonstrations from an optimal policy  $\pi^*$
- Recover a reward function *R* that makes  $\pi^*$  optimal
- Ill-Posed Problem
  - Infinite number of reward functions that can make  $\pi^*$  optimal
    - Trivial all zero reward
    - Constant reward
    - aR + c (positive scaling a>0, and affine shifts)

#### Simpler problem: What if you know the policy?

#### How would you do this more generally?

#### Basic IRL Algorithm

- Start with demonstrations, D
- Guess initial reward function  $R_0$
- $\hat{R} = R_0$
- Loop:
  - Solve for optimal policy  $\pi_{\widehat{R}}^*$
  - Compare *D* and  $\pi_{\hat{R}}^*$
  - Update  $\hat{R}$  to try and make D and  $\pi_{\hat{R}}^*$  more similar

### Feature count matching

• Assume the reward function is a linear combination of features:

$$R(s) = \mathbf{w}^T \phi(s)$$

• Value function becomes linear combination of (discounted) feature expectations:

$$V_R^{\pi} = \mathbb{E}_{\pi} \left[ \sum_{t=0}^{\infty} \gamma^t R(s_t) \right]$$

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$$V_R^{\pi} = \mathbf{w}^T \mathbb{E}_{\pi} \left[ \sum_{t=0}^{\infty} \gamma^t \phi(s_t) \right] = \mathbf{w}^T \mu_{\pi}$$

## Inverse reinforcement learning: feature matching (Abbeel and Ng 2004, Syed and Schapire 2007)

• If  $||w||_1 \le 1$ , then  $|x^{\top}y| \le ||x||_1 ||y||_{\infty}$ 

$$V_R^{\pi^*} - V_R^{\pi_{\text{robot}}} = \mathbf{w}^T (\mu_{\pi^*} - \mu_{\pi_{\text{robot}}})$$
$$\leq \|\mu_{\pi^*} - \mu_{\pi_{\text{robot}}}\|_{\infty}$$

- If feature expectations match, then expected returns are identical.
- Idea: Can we update the reward guess  $\hat{R}$  so the feature counts get closer?

## Problem: Many different policies can lead to same expected feature counts

### Maximum Entropy IRL (Ziebart et al. 2008)

 $P(\tau) = \frac{e^{R_w(\tau)}}{Z}$ 

 $R(s) = \mathbf{w}^T \phi(s)$ 

- Collect M demonstrations  $D = \{\tau_1, ..., \tau_M\}$
- Initialize reward weights **w**
- Loop
  - Solve for (soft) optimal policy  $\pi(a|s)$  via Value Iteration
  - Solve for expected feature counts of  $\pi(a|s)$
  - Compute weight update  $w \leftarrow w + \alpha(\mu_D \mu_\pi)$

#### Soft Value Iteration

$$\pi_{\Theta} \left( A_t | S_t \right) = e^{Q_{\pi_{\Theta}}^{\text{soft}}(A_t, S_t) - V_{\pi_{\Theta}}^{\text{soft}}(S_t)}$$
$$V_{\pi_{\Theta}}^{\text{soft}} \left( S_t \right) = \log \sum_{A_t \in \mathcal{A}} e^{Q_{\pi_{\Theta}}^{\text{soft}}(A_t, S_t)}$$
Soft Maximum

Policy is a softmax policy.

#### Softmax is a Soft Maximum

- Assume b > a
- $\log(e^a + e^b) =$
- If a = b
- $\log(e^a + e^b) =$

• In general  $\max\{x_1, x_2, \dots, x_n\} \le \log \sum_i \exp(x_i) \le \max\{x_1, \dots, x_n\} + \log n$ 

#### Soft Value Iteration

- Initialize values
- Repeat until convergence:
  - Solve for Q
  - Solve for V

#### Watch This: Scalable Cost-Function Learning for Path Planning in Urban Environments

Markus Wulfmeier<sup>1</sup>, Dominic Zeng Wang<sup>1</sup> and Ingmar Posner<sup>1</sup>







Fig. 1: Schema for training neural networks in the Maximum Entropy paradigm for IRL.

#### Another way to look at MaxEnt IRL

$$P(\tau) = \frac{e^{R_w(\tau)}}{Z} \qquad Z = \int e^{R_w(\tau)} d\tau$$

- Maximum Likelihood Estimation
- Find reward function that maximizes the log likelihood of the demonstration trajectories:

$$\max_{\theta} \frac{1}{N} \sum_{\tau \in D} R_w(\tau) - \log Z$$

#### How to avoid fully solving MDP

$$\max_{\theta} \frac{1}{N} \sum_{\tau \in D} R_w(\tau) - \log Z \qquad Z = \int e^{R_w(\tau)} d\tau$$

- Estimate Z with a finite set of trajectories  $Z_{\tau}$ .
- Loop:
  - Update parameters w so demonstrations have higher reward than trajectories in  $Z_{\tau}$ .
  - Update  $Z_{\tau}$

#### How to make this more tractable

**Relative Entropy Inverse Reinforcement Learning** 

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Uniform sampling to approximate Z.

 $P(\tau)$ 

#### Learning Objective Functions for Manipulation

Mrinal Kalakrishnan<sup>\*</sup>, Peter Pastor<sup>\*</sup>, Ludovic Righetti<sup>\*†</sup>, and Stefan Schaal<sup>\*†</sup> kalakris@usc.edu, pastorsa@usc.edu, ludovic.righetti@a3.epfl.ch, sschaal@usc.edu \*CLMC Lab, University of Southern California, Los Angeles CA 90089 <sup>†</sup>Max Planck Institute for Intelligent Systems, Tübingen, Germany 72076

Guided Cost Learning: Deep Inverse Optimal Control via Policy Optimization

Chelsea Finn Sergey Levine Pieter Abbeel University of California, Berkeley, Berkeley, CA 94709 USA CBFINN@EECS.BERKELEY.EDU SVLEVINE@EECS.BERKELEY.EDU PABBEEL@EECS.BERKELEY.EDU Noisy perturbations of demonstrations to approximate Z

Use current policy to approximate Z. Alternate between a few steps of reward updates and a few steps of policy updates.

 $e^{R_W(\tau)}$ 

#### Finn et al. "Guided Cost Learning." 2016



#### GANs (Generative Adversarial Networks)




#### GAIL (Generative Adversarial Imitation Learning)



Ho and Ermon, 2016

# What if we don't want just a single reward estimate?

• Can we get a samples from the full Bayesian posterior?

# $P(R|D) \propto P(D|R)P(R)$

#### Bayesian Inverse Reinforcement Learning (Ramachandran and Amir 2007)

- Assume demonstrator is Boltzmann rational
  - Demonstrator follows a softmax policy with inverse temperature c

$$P(D|R) = \prod_{(s,a)\in D} \frac{e^{\beta Q^*(s,a,R)}}{\sum_{b\in A} e^{\beta Q^*(s,b,R)}}$$

 $Q^*(s, a, R) = {
m How \ much \ reward \ will \ I \ expect \ to \ see \ if \ I \ take \ action} \ a \ in \ state \ s \ and \ act \ optimally \ thereafter.$ 

#### Bayesian Inverse Reinforcement Learning (Ramachandran and Amir 2007)

- Assume demonstrator is Boltzmann rational
  - Demonstrator follows a softmax policy with inverse temperature  $\beta$

$$\begin{split} P(D|R) &= \prod_{(s,a)\in D} \underbrace{\frac{e^{\beta Q^*(s,a,R)}}{\sum_{b\in A} e^{\beta Q^*(s,b,R)}}}_{\substack{b\in A} e^{\beta Q^*(s,b,R)}} \end{split}$$

#### Bayesian Inverse Reinforcement Learning (Ramachandran and Amir 2007)

- Assume demonstrator is Boltzmann rational
  - Demonstrator follows a softmax policy with inverse temperature  $\beta$

$$P(D|R) = \prod_{(s,a)\in D} \frac{e^{\beta Q^*(s,a,R)}}{\sum_{b\in A} e^{cQ^*(s,b,R)}}$$

Perform Bayesian inference (MCMC) to sample from posterior distribution

$$P(R|D) \propto P(D|R)P(R)$$



# Applications of Bayesian IRL

- Active Learning
- Uncertainty Estimation
- Demonstration Sufficiency





Center for Human-Compatible Artificial Intelligence



#### Autonomous Assessment of Demonstration Sufficiency via Bayesian Inverse Reinforcement Learning

Tu (Alina) Trinh University of California, Berkeley Haoyu Chen Daniel S. Brown University of Utah University of Utah









## Learning From Demonstration (LfD)





- Have I provided enough demonstrations?
- Are my demonstrations informative enough?
- Should I just supervise the robot?



- Have I received enough demonstrations?
- Are these demonstrations informative enough?

**Demonstration Insufficiency** 

**Demonstration Insufficiency** 



**Uninformative Demos** 

**Demonstration Insufficiency** 



**Demonstration Insufficiency** 



**Demonstration Sufficiency** 

**Demonstration Sufficiency** 



Demos

**Demonstration Sufficiency** 



**Demonstration Sufficiency** 





- Have I received enough demonstrations?
- Are these demonstrations informative enough?
- What is the reward function?
- How do I measure policy "goodness"?









# Measuring Policy Goodness

• Normalized expected value difference (**nEVD**)

$$nEVD(\pi_{\text{robot}}, R^*) = \frac{V_{R^*}^* - V_{R^*}^{\pi_{\text{robot}}}}{V_{R^*}^* - V_{R^*}^{\pi_{\text{rand}}}}$$

• Puts policy regret in interpretable percentage form

# Measuring Policy Goodness

• Normalized expected value difference (nEVD)

$$nEVD(\pi_{\text{robot}}, R^*) = \frac{V_{R^*}^* - V_{R^*}^{\pi_{\text{robot}}}}{V_{R^*}^* - V_{R^*}^{\pi_{\text{rand}}}}$$

- Puts policy regret in interpretable percentage form
- We only have an estimate of R\*, so...







# Comparing With Theoretical Bounds

How many demonstrations is enough for a simple gridworld?

nEVD Threshold	Ours	Abbeel and Ng '04	Syed and Schapire '07
0.1	17	1,600,000	3,700,000
0.3	16	180,000	410,000
0.5	15	65,000	150,000
			(

based on Chernoff-Hoeffding bound

[1] Pieter Abbeel and Andrew Y Ng. Apprenticeship learning via inverse reinforcement learning. ICML 2004.[2] Umar Syed and Robert E Schapire. A game-theoretic approach to apprenticeship learning. NeurIPS 2007.

#### **Baselines and Environments**

- Convergence heuristic
- Validation set heuristic

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- Convergence heuristic
- Validation set heuristic



#### Simulation Results

Ours Convergence Valid. Set









#### User Study Results


## User Study Results



robust against suboptimal demonstrations: ~12%

### Future Work













### Autonomous Assessment of Demonstration Sufficiency via Bayesian Inverse Reinforcement Learning



## What if I want to learn rewards from more than demonstrations?

#### **Reward-rational (implicit) choice:** A unifying formalism for reward learning

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## How do we learn from diverse types of feedback of unknown quality?







ERGEN

## Unifying Human Feedback Types

Boltzmann Rational Choice Model  $C = \{c_1, \dots, c_n\} \quad r: C \to \mathbb{R}$   $\mathbb{P}(c_i | r) = \frac{\exp(\beta r(c_i))}{\sum_C \exp(\beta r(c'))}$ 

#### $\beta \rightarrow 0$ random (non-rational) choices

 $\beta \rightarrow \infty$  deterministic (perfectly-rational) choices

Jeon et al. "Reward-rational (implicit) choice: A unifying formalism for reward learning." NeurIPS 2020.

## Trajectory Comparisons (Pairwise Prefs)

 $\tau_2$   $\tau_1$ 

 $C = \{\tau_1, \tau_2\}$ 

Boltzmann Rational Choice Model  $C = \{c_1, \dots, c_n\} \quad r: C \to \mathbb{R}$   $\mathbb{P}(c_i | r) = \frac{\exp(\beta r(c_i))}{\sum_C \exp(\beta r(c'))}$ 

$$\mathbb{P}(\tau_i|r) = \frac{\exp(\beta r(\tau_i))}{\exp(\beta r(\tau_i)) + \exp(\beta r(\tau_i))}$$

### Demonstrations

C = {All Possible Trajectories }

$$\mathbb{P}(\tau|r) = \frac{\exp(\beta r(\tau))}{\sum_{\mathrm{T}} \exp(\beta r(\tau'))}$$

Boltzmann Rational Choice Model  $C = \{c_1, \dots, c_n\} \quad r: C \to \mathbb{R}$   $\mathbb{P}(c_i | r) = \frac{\exp(\beta r(c_i))}{\sum_C \exp(\beta r(c'))}$ 

## E-Stops



$$\mathbb{P}(\tau_{:i}|r) = \frac{\exp(\beta r(\tau_{:i}))}{\sum_{1}^{T} \exp\beta r(\tau_{:j}))}$$

## Reward Learning from Human Feedback

- Assume demonstrator is Boltzmann rational
  - Demonstrator follows a softmax policy with inverse temperature  $\beta$

$$C = \{c_1, \dots, c_n\} \quad r: C \to \mathbb{R}$$
$$\mathbb{P}(c_i | r) = \frac{\exp(\beta r(c_i))}{\sum_C \exp(\beta r(c'))}$$

Perform Bayesian inference (MCMC) to sample from posterior distribution

$$P(R|D) \propto P(D|R)P(R)$$



## RL from Human Feedback (RLHF)



### RL from Human Preferences



https://arxiv.org/abs/1706.03741

# Why would you want to learn a reward from ranked examples?

### Inverse Reinforcement Learning

Prior approaches ...

1. Typically couldn't do much better than the demonstrator.

2. Were hard to scale to complex problems.

#### Pre-Ranked Demonstrations



## Inverse Reinforcement Learning

Prior approaches ...

Pre-Ranked Demonstrations

- Typically couldn't do much better than the demonstrator.
- Find a reward function that explains the ranking, allowing for extrapolation.
- 2. Were hard to scale to complex problems.



## Inverse Reinforcement Learning

Prior approaches ...

Pre-Ranked Demonstrations

 Typically couldn't do much better than the demonstrator.

Find a reward function that explains the ranking, allowing for extrapolation.

2. Were hard to scale to complex problems.

Reward learning becomes a supervised learning problem.



## Trajectory-ranked Reward Extrapolation (T-REX)



#### Pre-ranked demonstrations

## Trajectory-ranked Reward Extrapolation (T-REX)



#### **Pre-ranked demonstrations**

T-REX Policy

### **Reward Function**

 $R_{\theta}: S \to \mathbb{R}$ 

#### Examples of S:

Current Robot Joint Angles and Velocities

$$\boxed{\swarrow} \rightarrow 0.5 \qquad \boxed{\checkmark} \rightarrow -0.7$$

### **Reward Function**

 $R_{\theta}: S \to \mathbb{R}$ 

#### Examples of S:

Current Robot Joint Angles and Velocities

> Short Sequence of Images



## Trajectory-ranked Reward Extrapolation (T-REX) $\tau_1 \prec \tau_2 \prec \cdots \prec \tau_T$

$$\sum_{s \in \tau_1} R_{\theta}(s) < \sum_{s \in \tau_2} R_{\theta}(s)$$

Bradley-Terry pairwise ranking loss

$$\exp\sum_{s\in\tau_j}R_\theta(s)$$

$$\mathcal{L}(\theta) = -\sum_{\tau_i \prec \tau_j} \exp\sum_{s \in \tau_i} R_{\theta}(s) + \exp\sum_{s \in \tau_j} R_{\theta}(s)$$





#### **T-REX Policy Performance**



## **Reward Extrapolation**



#### T-REX can extrapolate beyond the performance of the best demo

## "Autonomous Driving" in Atari





Best demo (Score = 84)

**T-REX (Score = 520)** 

#### **Uses only 12 ranked demonstrations**

### Atari Breakout



# What if you don't have explicit preference labels?

Learning from a learner [ICML'19]



#### Automatic preference label generation [CoRL'20]



## Automatic Rankings via Noise Injection

- Assumption: Demonstrator is significantly better than a purely random policy.
- Provides automatic rankings as noise increases.
- Generates a large diverse set of ranked demonstrations



Brown et al. "Better-than-Demonstrator Imitation Learning via Automatically-Ranked Demonstrations." CoRL 2019

# Disturbance-based Reward Extrapolation (D-REX)



3-72



Brown et al. "Better-than-Demonstrator Imitation Learning via Automatically-Ranked Demonstrations." CoRL 2019

# Disturbance-based Reward Extrapolation (D-REX)





# Disturbance-based Reward Extrapolation (D-REX)







## Experiments

D-REX consistently outperforms the best demonstration as well as outperforming BC and GAIL.



Brown et al. "Better-than-Demonstrator Imitation Learning via Automatically-Ranked Demonstrations." CoRL 2019




AI systems can **efficiently** infer human intent from **suboptimal demonstrations**.

# T-REX only learns a maximum likelihood estimate of the reward function.



## Reward Hacking





- Overfit to spurious correlations
- No consideration of alternative hypotheses



#### Idea: Fast Bayesian Inference



Brown et al. "Safe Imitation Learning via Fast Bayesian Reward Inference from Preferences." ICML 2020. <sup>125</sup>

### Next time: LLMs and ChatGPT



#### **Prompts Dataset**

