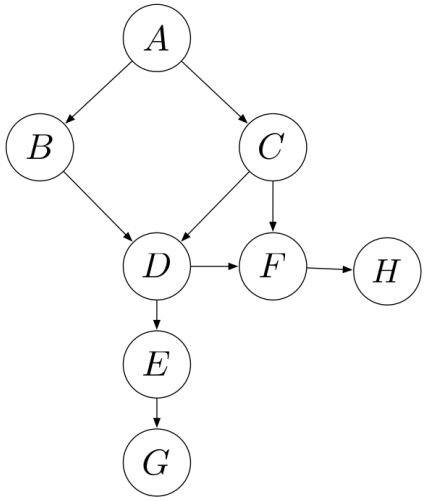
1 D-Separation

For the Bayes net below, determine if each independence assertion is guaranteed to be true.



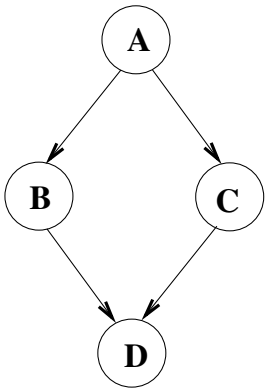
1. $B \perp\!\!\!\perp C$
2. $B \perp\!\!\!\perp C \mid G$
3. $B \perp\!\!\!\perp C \mid H$
4. $A \perp\!\!\!\perp D \mid G$
5. $A \perp\!\!\!\perp D \mid H$
6. $B \perp\!\!\!\perp C \mid A, F$
7. $F \perp\!\!\!\perp B \mid D, A$
8. $F \perp\!\!\!\perp B \mid D, C$

2 Sampling

Consider the Bayes net below with corresponding CPTs.

1. Using Prior Sampling, generate 2 samples using the following random numbers for the following nodes:

A	B	C	D	A	B	C	D
0.31	0.58	0.04	0.94	0.67	0.49	0.37	0.12



A	P(A)
+a	0.8
-a	0.2

A	B	$P(B\|A)$
+a	+b	0.8
+a	-b	0.2
-a	+b	0.5
-a	-b	0.5

A	C	$P(C\|A)$
+a	+c	0.7
+a	-c	0.3
-a	+c	0.1
-a	-c	0.9

B	C	D	$P(D\|B,C)$
+b	+c	+d	0.3
+b	+c	-d	0.7
+b	-c	+d	0.1
+b	-c	-d	0.9
-b	+c	+d	0.2
-b	+c	-d	0.8
-b	-c	+d	0.9
-b	-c	-d	0.1

The samples are:

Sample 1:
Sample 2:

2. Given the samples below, answer the subsequent queries. We've done the first one for you.

+a -b -c -d
-a +b +c -d
+a -b +c -d
+a +b -c -d
+a -b +c +d
-a -b -c +d
-a -b -c -d
+a +b +c -d
-a +b -c -d
+a +b -c +d

(a) $P(+d) = 3/10$

(b) $P(+a, -b) =$

(c) $P(-a, -b, -c, -d) =$

(d) $P(+d | -a, -b) =$

3. Consider the query $P(-d | -a, -b)$. Using the following random numbers, generate samples and their weights using likelihood weighting.

C	D	C	D
0.31	0.58	0.09	0.25

The samples and weights are:

Sample 1 and weight:
Sample 2 and weight

4. Given the weighted samples below, answer the subsequent queries. We've done the first one for you.

+a	-b	-c	-d	0.3
-a	+b	+c	-d	0.4
+a	-b	+c	-d	0.1
+a	+b	-c	-d	0.3
+a	-b	+c	+d	0.4
-a	-b	-c	+d	0.1
-a	-b	-c	-d	0.2
+a	+b	+c	-d	0.5
-a	+b	-c	-d	0.7
+a	+b	-c	+d	0.8

(a) $P(+d) = (0.4+0.1+0.8)/(0.3+0.4+0.1+0.3+0.4+0.1+0.2+0.5+0.7+0.8) = 1.3/3.8$

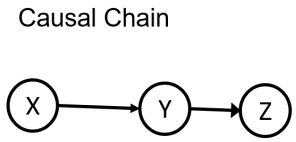
(b) $P(+a, -b) =$

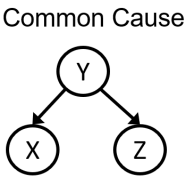
(c) $P(-a, -b, -c, -d) =$

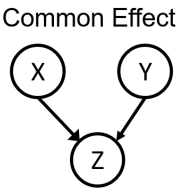
(d) $P(+d | -a, -b) =$

3 (6300 only) D-Separation Proofs

1. Write out formal proofs for the following. Don't just write a bunch of equations. Make sure that you justify each step in words, provide a high-level explanation of what you are doing and why, and include a full derivation showing all your steps.







(a) Causal Chain: $X \perp\!\!\!\perp Z \mid Y$

(b) Common Cause: $X \perp\!\!\!\perp Z \mid Y$

(c) Common Effect: $X \perp\!\!\!\perp Y$

2. Draw the topology for 4 different Bayes' nets that all obey the property that $X \perp\!\!\!\perp Y \mid W$. Each Bayes' net should have four random variables. Call them X, Y, Z, and W.