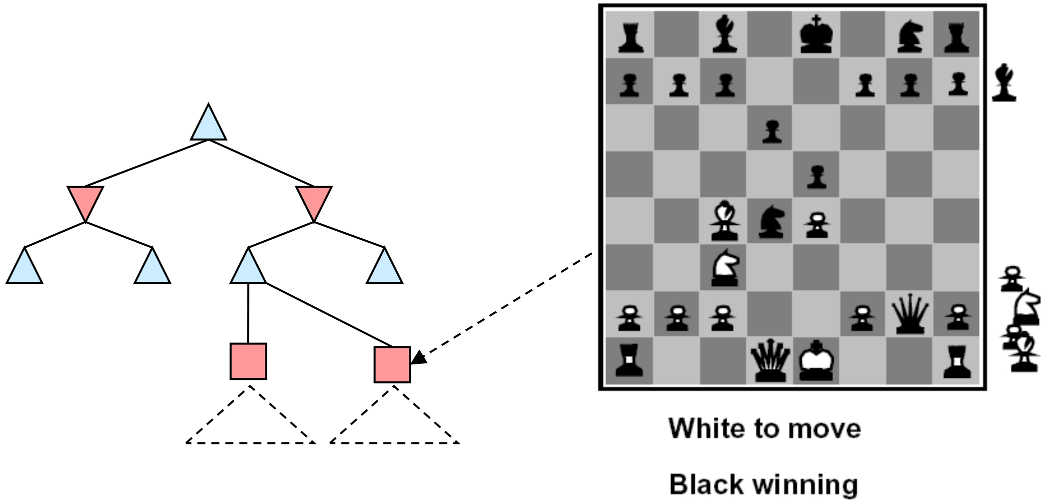
Final Exam Review

- Friday 10:30-12:30pm in class!
- Bring a calculator. You can check out one from the library.
- One page of notes front and back.

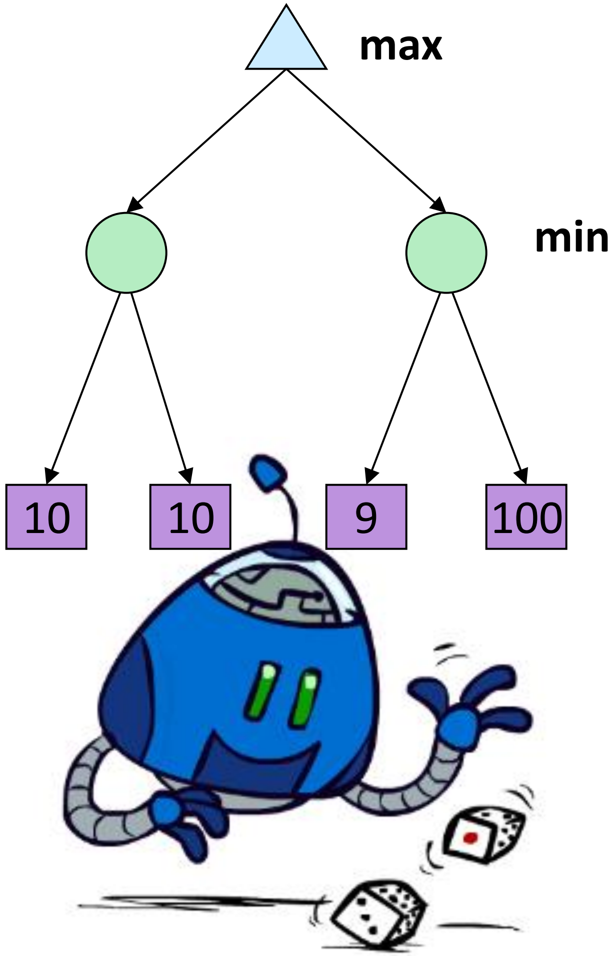
Look where we've been!

Informed Search:
A-Star

Adversarial Search:
Alpha-Beta Pruning

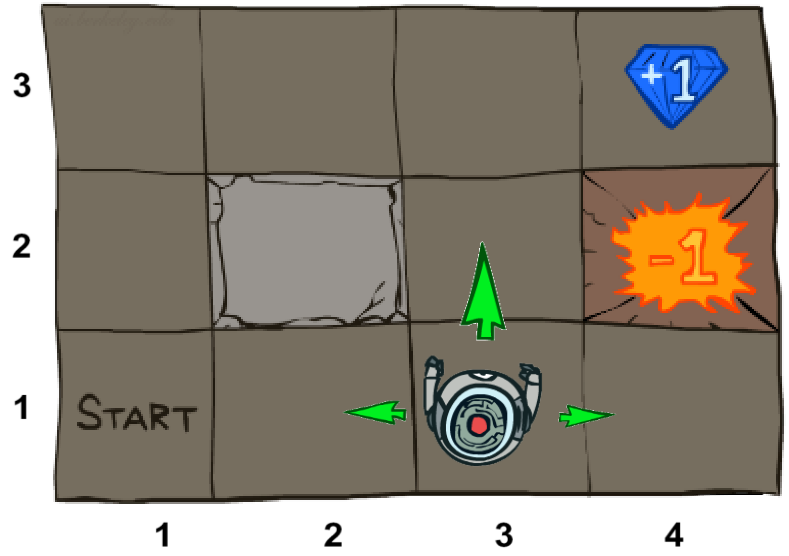


Expectimax Search

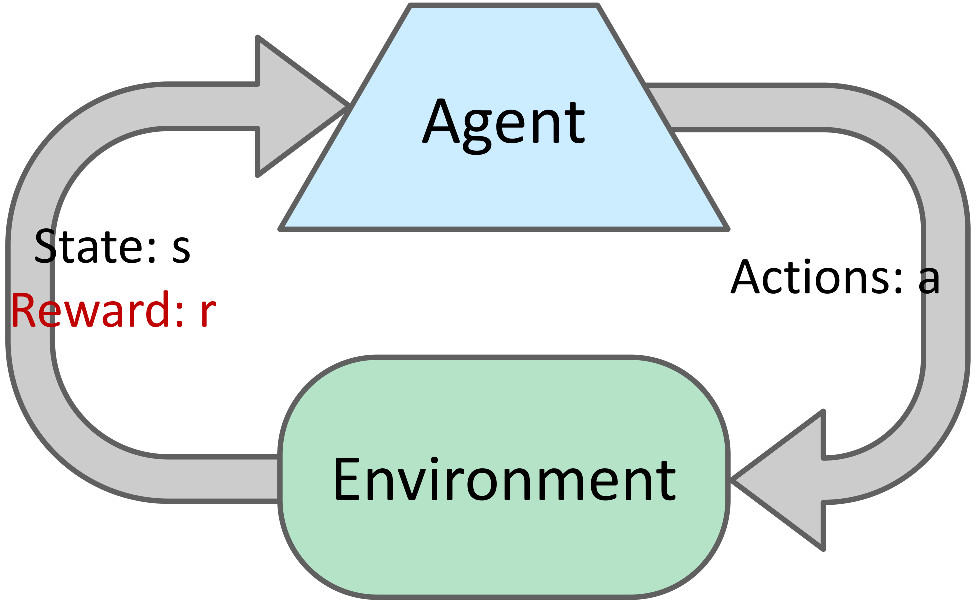


Look where we've been!

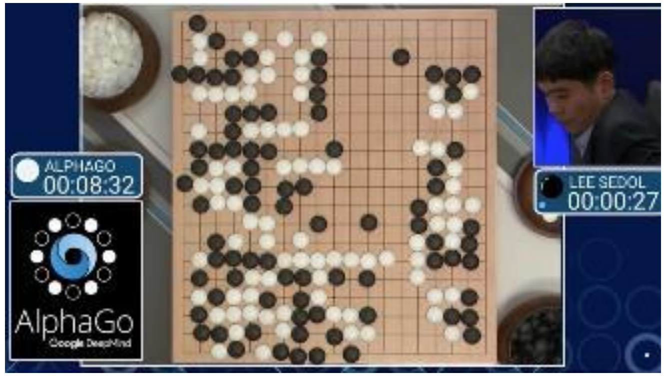
MDPs:
Value Iteration, Policy Iteration



Reinforcement Learning: Q-Learning, Policy Gradients

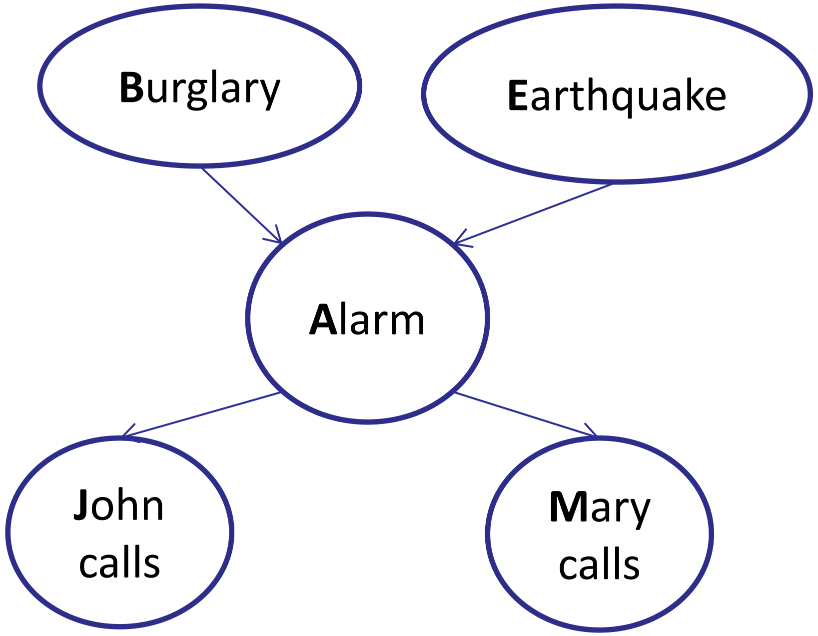


DQN
AlphaGo

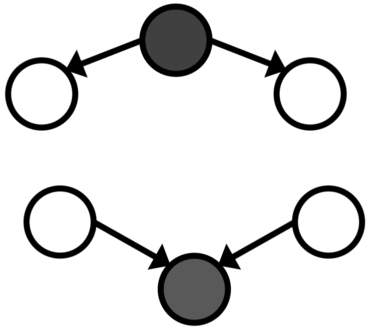


Look where we've been!

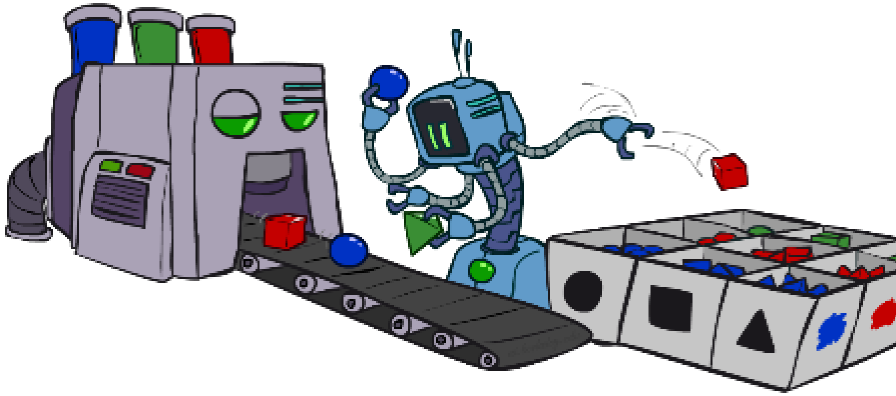
Bayes' Nets



D-Separation

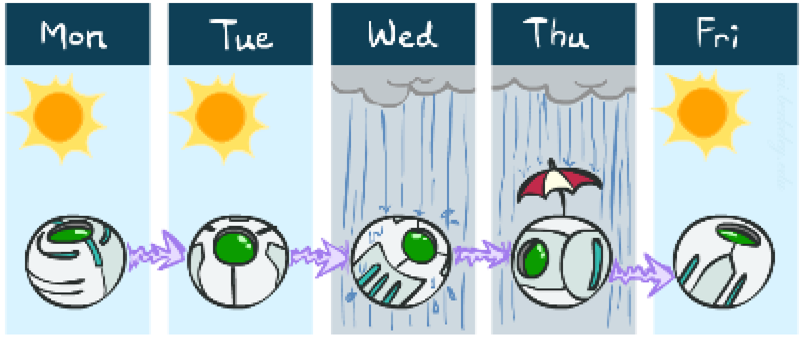


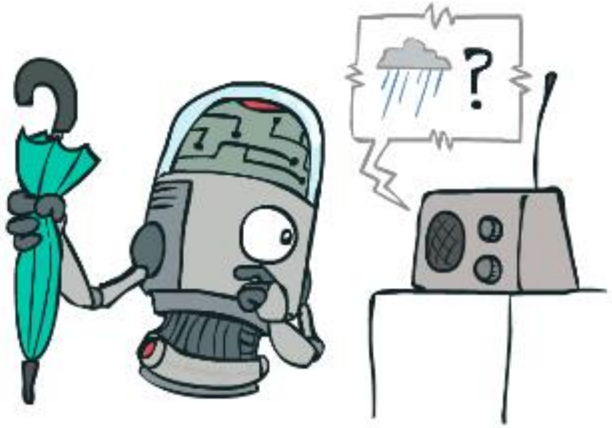
Sampling



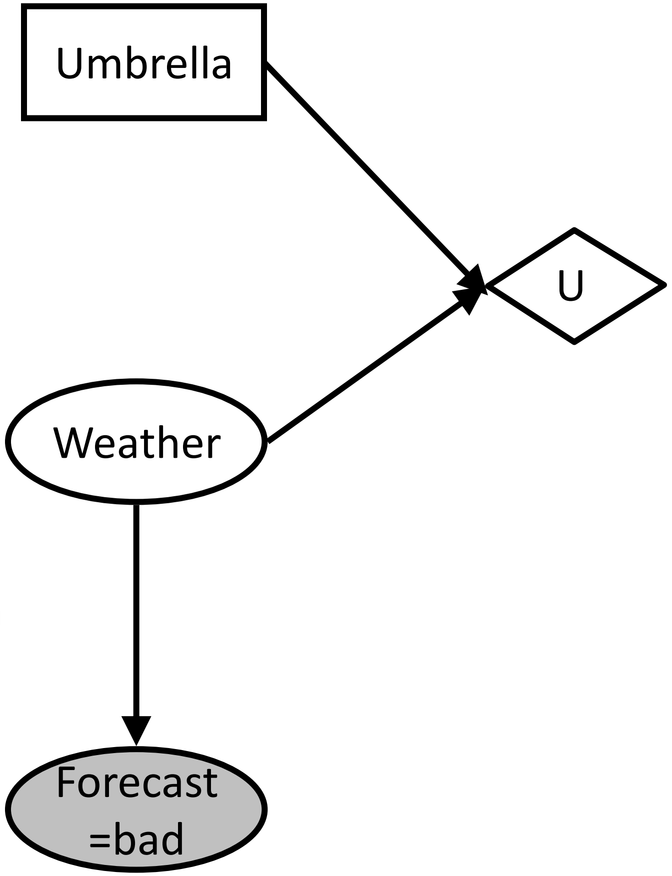
Look where we've been!

Markov Models

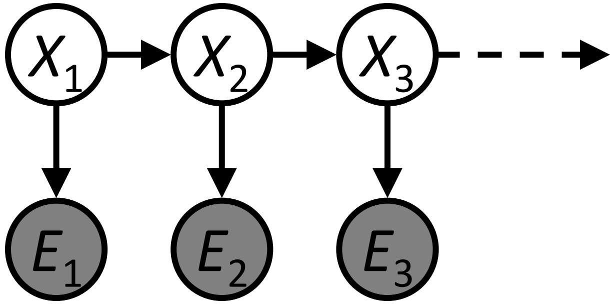


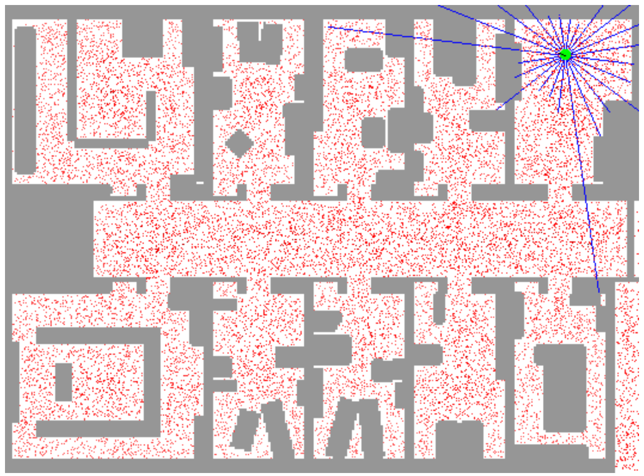


Value of Perfect Information



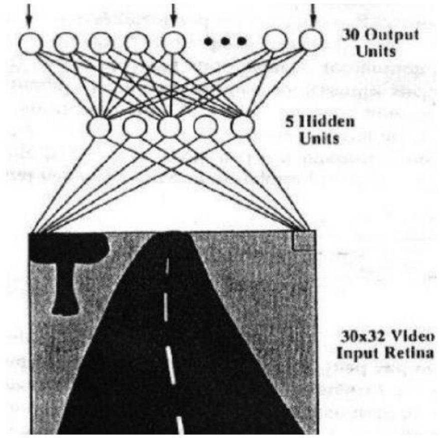
Hidden Markov Models:
Particle Filters



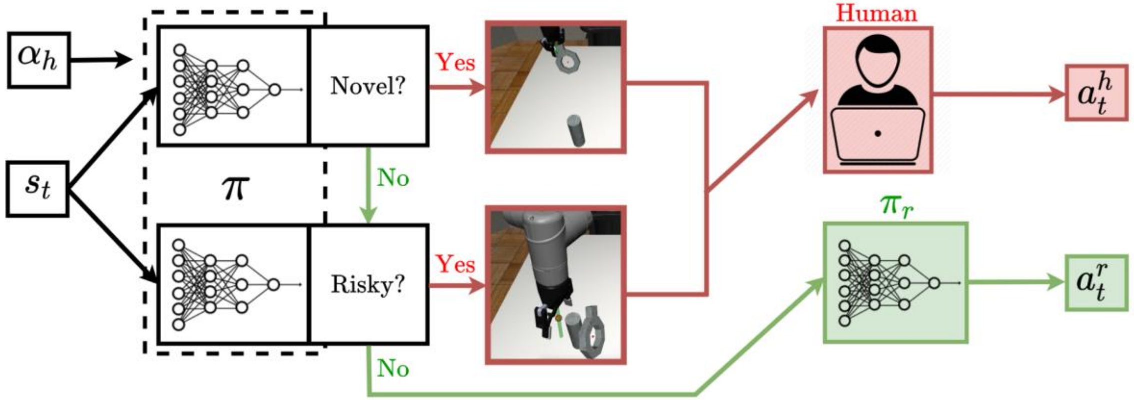


Look where we've been!

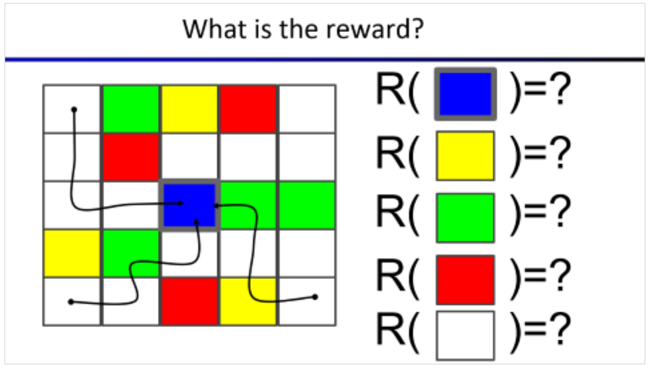
Behavioral Cloning



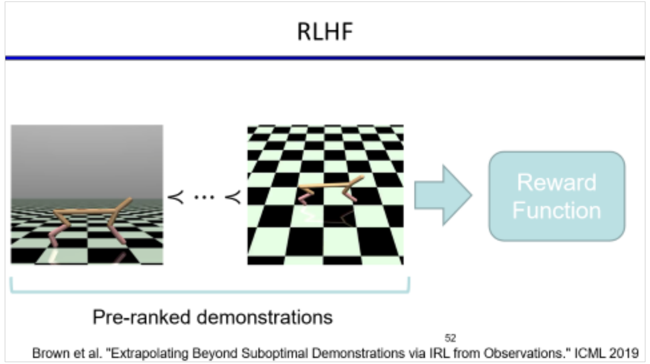
DAgger



Inverse RL

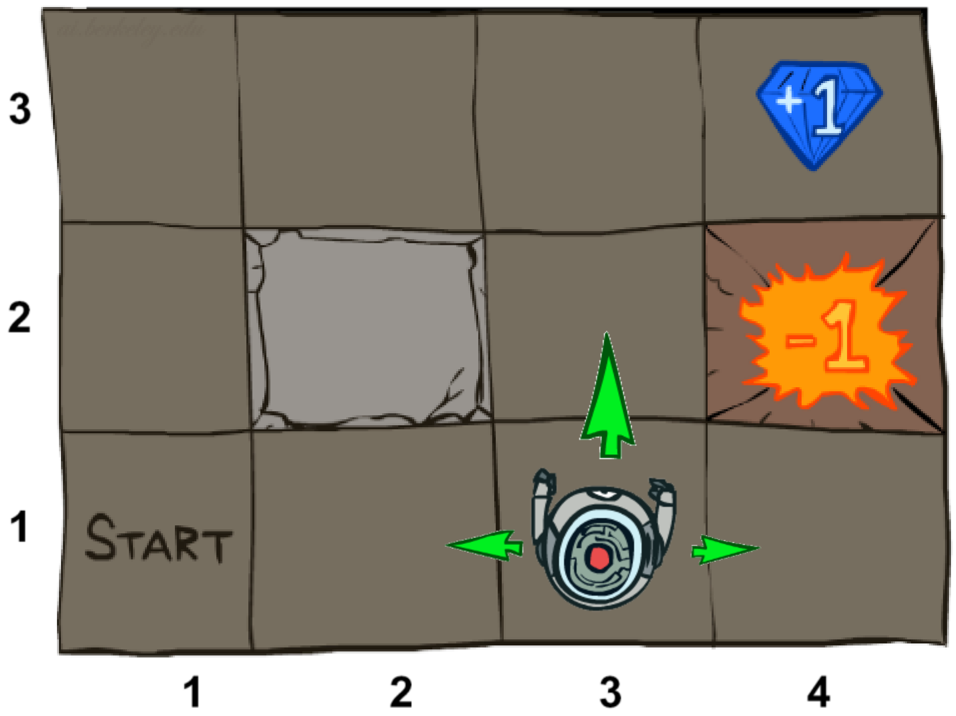


RL from Human Feedback



Markov Decision Processes

- An MDP is defined by:
  - A set of states s ∈ S
  - A set of actions a ∈ A
  - A transition function T(s, a, s')
    - Probability that a from s leads to s', i.e., P(s'| s, a)
    - Also called the model or the dynamics
  - A reward function R(s, a, s')
    - Sometimes just R(s) or R(s')
  - A start state
  - Maybe a terminal state

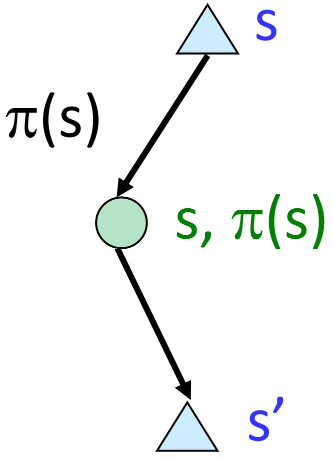


- MDPs are non-deterministic search problems
  - One way to solve them is with expectimax search
  - We'll have a new tool soon

[Demo – gridworld manual intro (L8D1)]

Temporal Difference Learning

- Big idea: learn from every experience!
  - Update V(s) each time we experience a transition (s, a, s', r)
  - Likely outcomes s' will contribute updates more often
- Temporal difference learning of values
  - Policy still fixed, still doing evaluation!
  - Move values toward value of whatever successor occurs: running average



Sample of V(s): $sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$

Update to V(s): $V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + (\alpha)sample$

Same update: $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(sample - V^{\pi}(s))$

Q-Learning

- Q-Learning: sample-based Q-value iteration

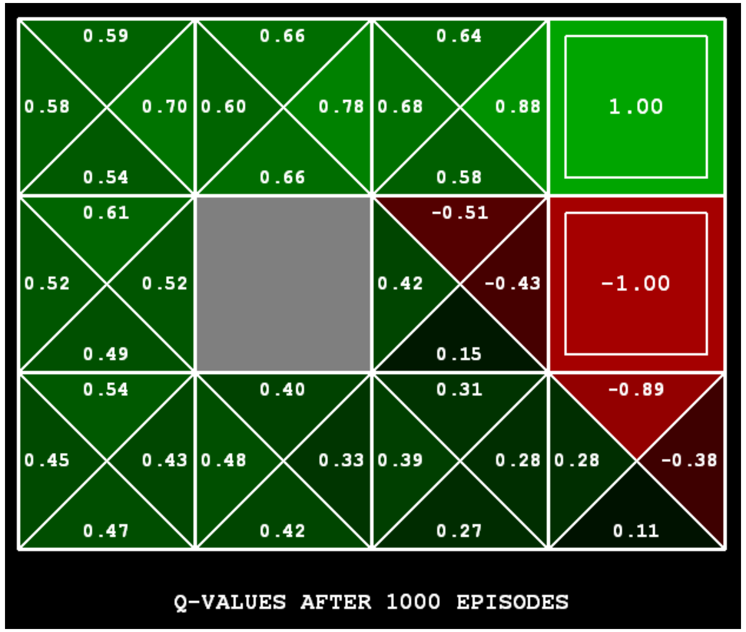
$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')\right]$$

- Learn Q(s,a) values as you go
  - Receive a sample (s,a,s',r)
  - Consider your old estimate: $Q(s,a)$
  - Consider your new sample estimate:

$$sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

  - Incorporate the new estimate into a running average:

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\left[sample\right]$$



Linear Value Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Advantage: our experience is summed up in a few powerful numbers

- Disadvantage: states may share features but actually be very different in value!

Approximate Q-Learning

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

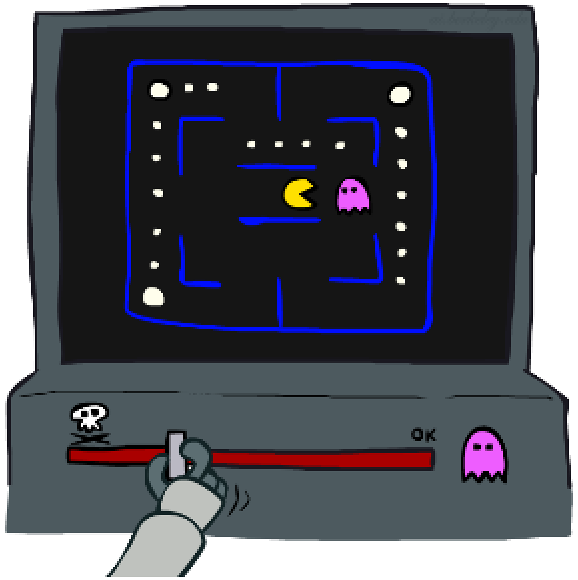
- Q-learning with linear Q-functions:

transition $= (s, a, r, s')$

difference $= \left[r + \gamma \max_{a'} Q(s', a') \right] - Q(s, a)$

$Q(s,a) \leftarrow Q(s,a) + \alpha$ [difference] Exact Q's

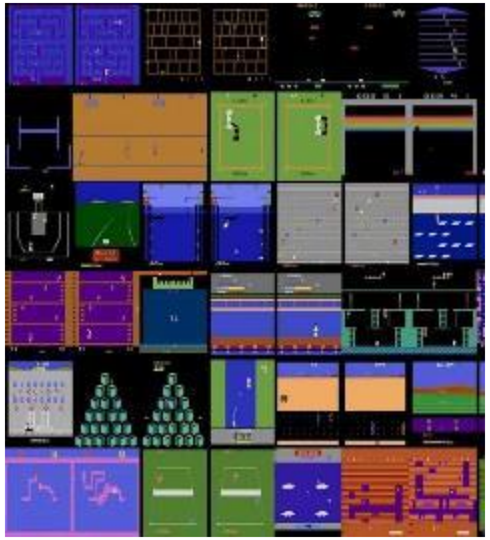
$w_i \leftarrow w_i + \alpha$ [difference] $f_i(s,a)$ Approximate Q's

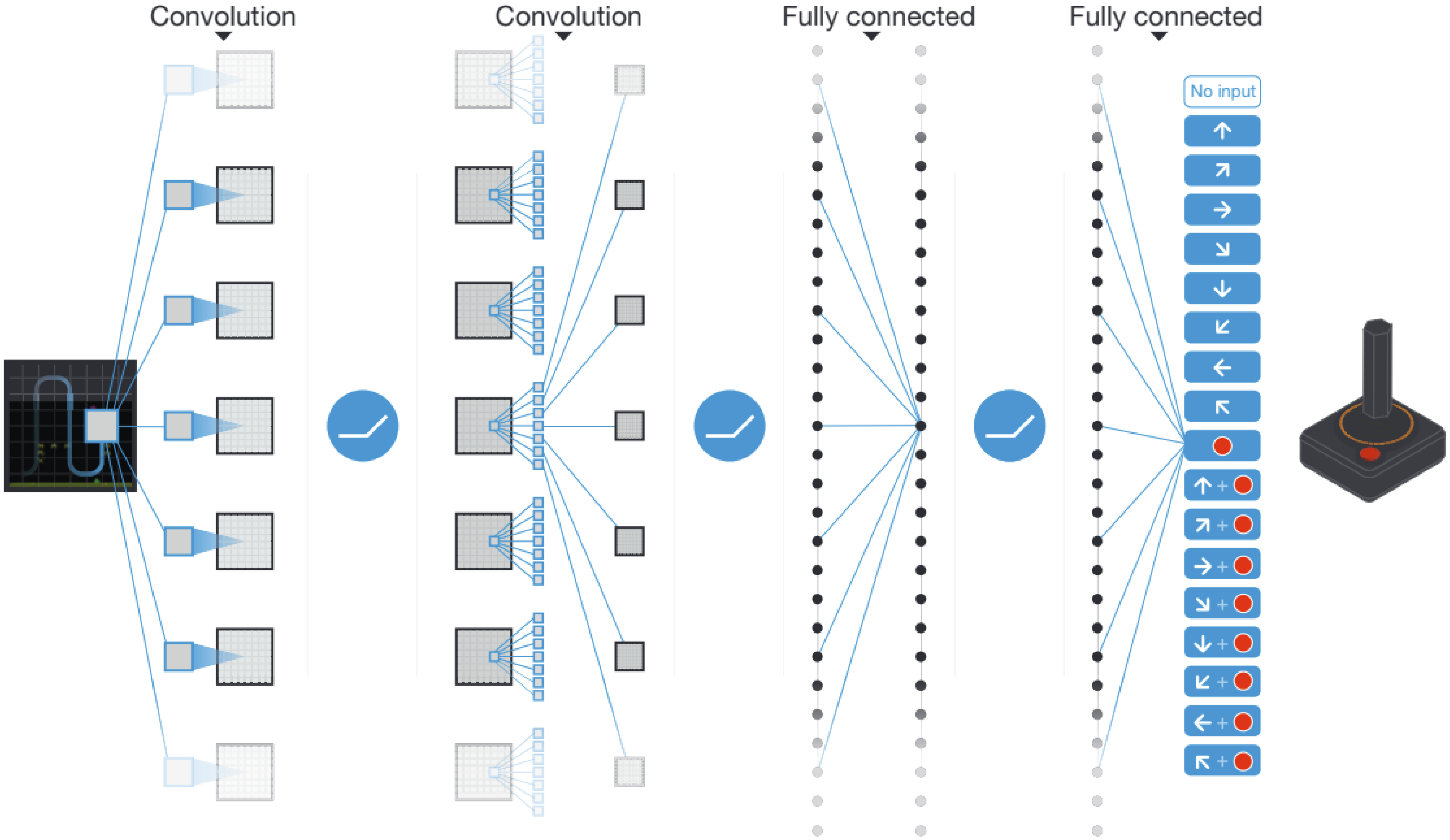


- Intuitive interpretation:
  - Adjust weights of active features
  - E.g., if something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features
- Formal justification: online least squares

DQN

- Approximate Q-Learning at scale.
- Uses Neural Network for Q-value function approximation.





Two approaches to model-free RL

- Learn Q-values
  - Trains Q-values to be consistent. Not directly optimizing for performance.
  - Use an objective based on the Bellman Equation

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

- Learn Policy Directly
  - Have a parameterized policy π_θ
  - Update the parameters θ to optimize performance of policy.

Policy Gradient RL

- We want a policy that maximizes expected utility (discounted cumulative rewards)
- We also want a way to learn with continuous action spaces

$$\pi^* = arg\max_{\pi} E_{\pi}\left[\sum_{t=0}^{\infty} \gamma^t R(s, \pi(s), s')\right]$$

The Policy Gradient

- We can now perform gradient ascent to improve our policy!

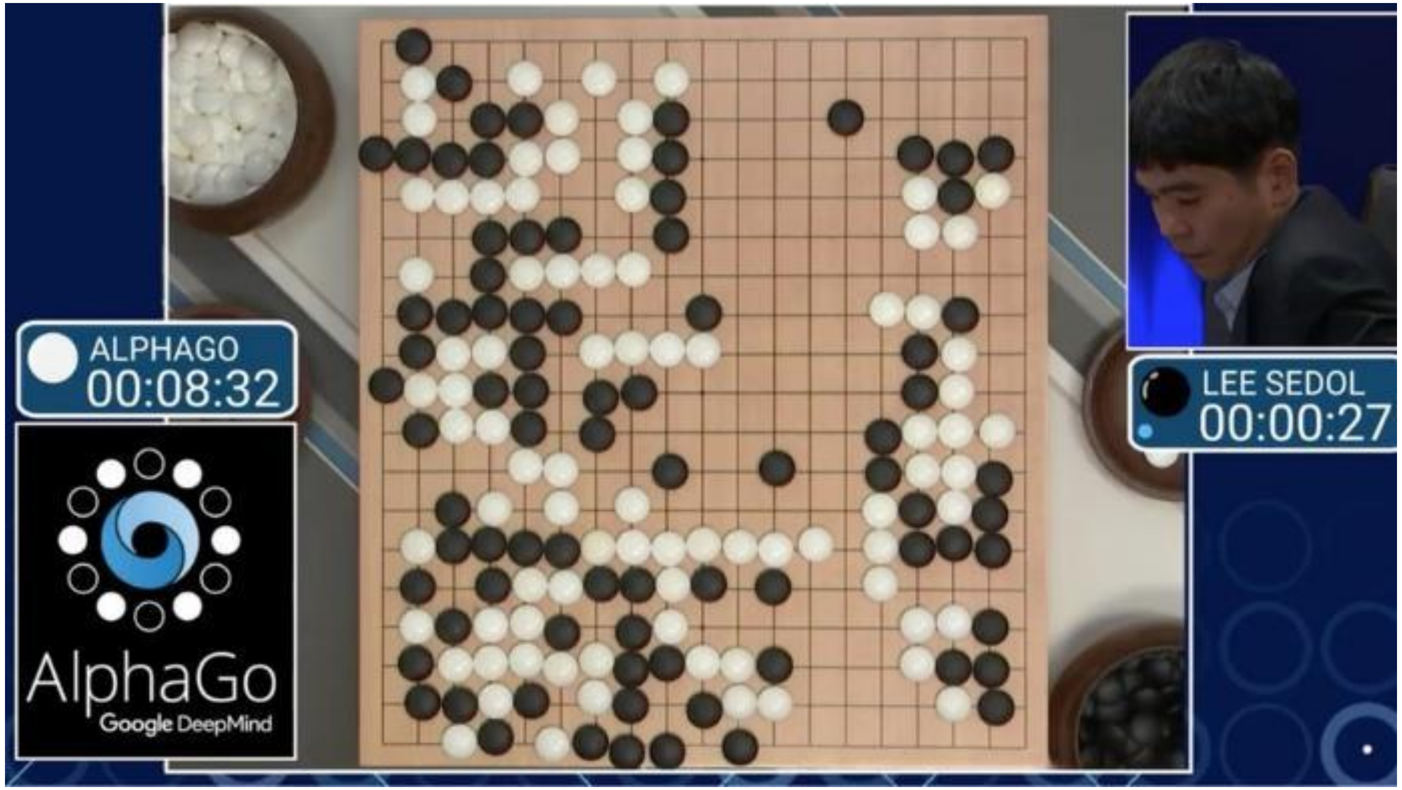
$$\theta_{k+1} \leftarrow \theta_k + \alpha \nabla_\theta J(\pi_\theta) \Big|_{\theta_k}$$

$$\nabla_\theta J(\pi_\theta) = E_{\tau \sim \pi_\theta}\left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t)\ R(\tau)\right]$$

Estimate with a sample mean over a set D of policy rollouts given current parameters

$$\approx \frac{1}{|D|} \sum_{\tau \in D} \sum_{t=0}^{T} (\nabla_\theta \log \pi_\theta(a_t|s_t)\ R(\tau))$$

Alpha Go



There will be one short answer question about AlphaGo.

Review high-level ideas from slides. Don't worry about nitty-gritty details.

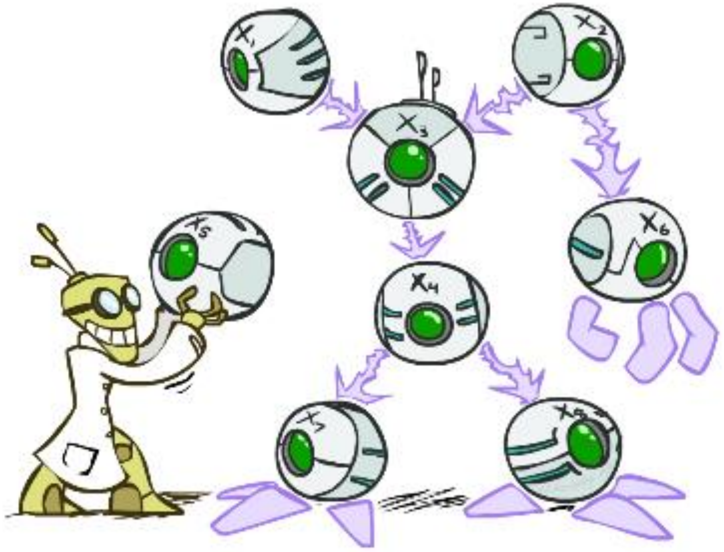
Bayes' Net Semantics

- A directed, acyclic graph, one node per random variable
- A conditional probability table (CPT) for each node
  - A collection of distributions over X, one for each combination of parents' values

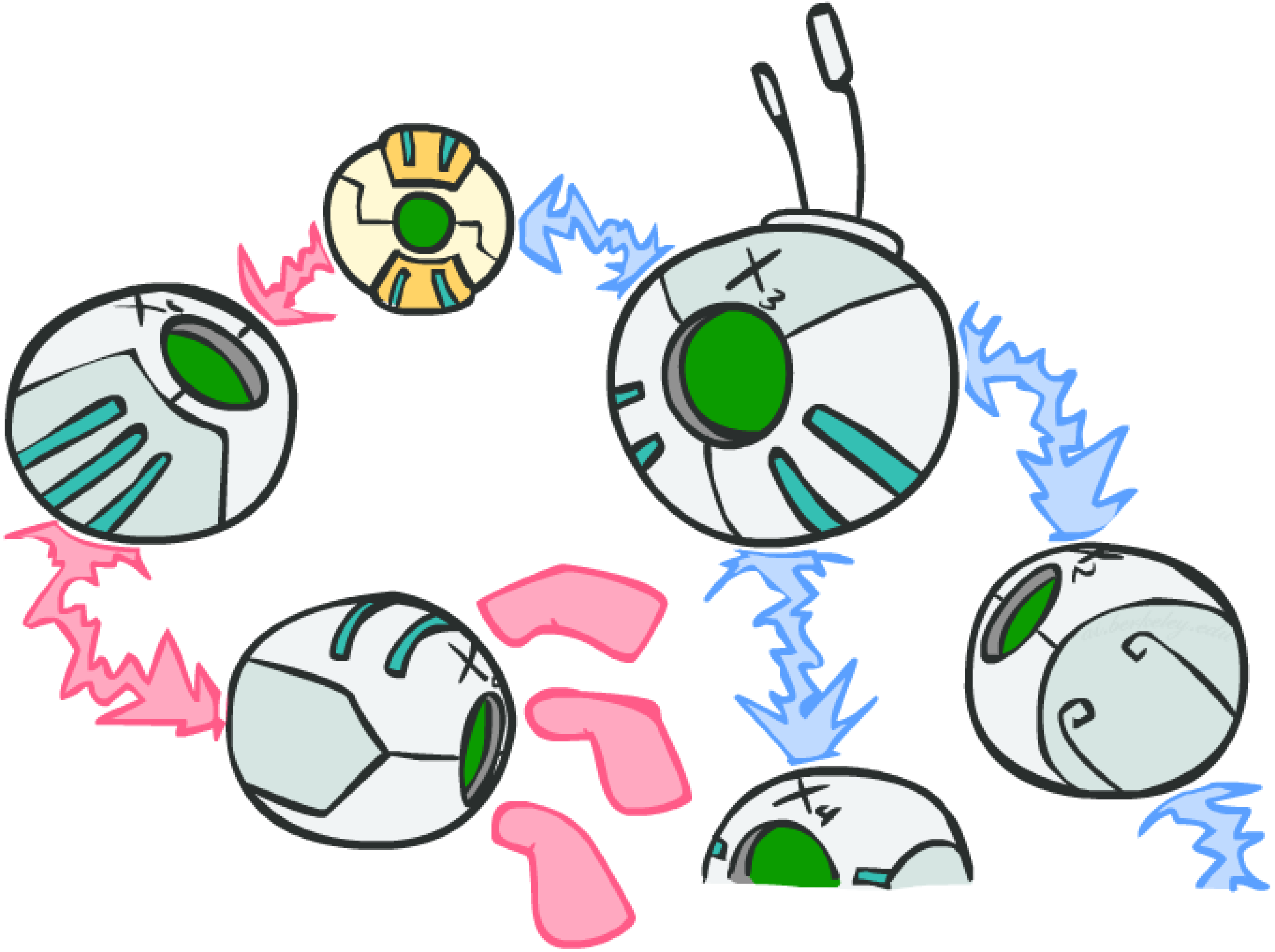
$$P(X|a_1 \ldots a_n)$$

- Bayes' nets implicitly encode joint distributions
  - As a product of local conditional distributions
  - To see what probability a BN gives to a full assignment, multiply all the relevant conditionals together:

$$P(x_1, x_2, \ldots x_n) = \prod_{i=1}^{n} P(x_i|parents(X_i))$$

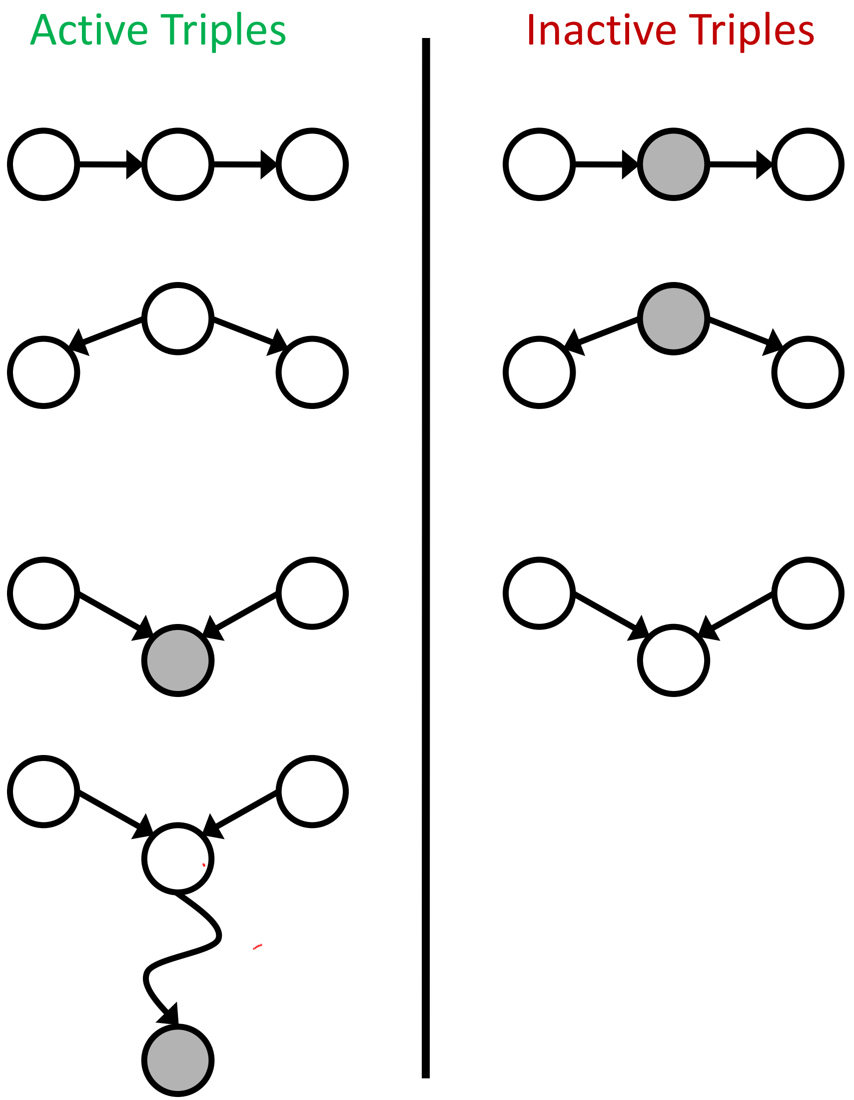


D-separation: Outline



Active / Inactive Paths

- Question: Are X and Y conditionally independent given evidence variables {Z}?
  - Yes, if X and Y “d-separated” by Z
  - Consider all (undirected) paths from X to Y
  - No active paths = independence!
- A path is active if each triple is active:
  - Causal chain A → B → C where B is unobserved (either direction)
  - Common cause A ← B → C where B is unobserved
  - Common effect (aka v-structure)
    A → B ← C where B *or one of its descendents* is observed
- All it takes to block a path is a single inactive segment



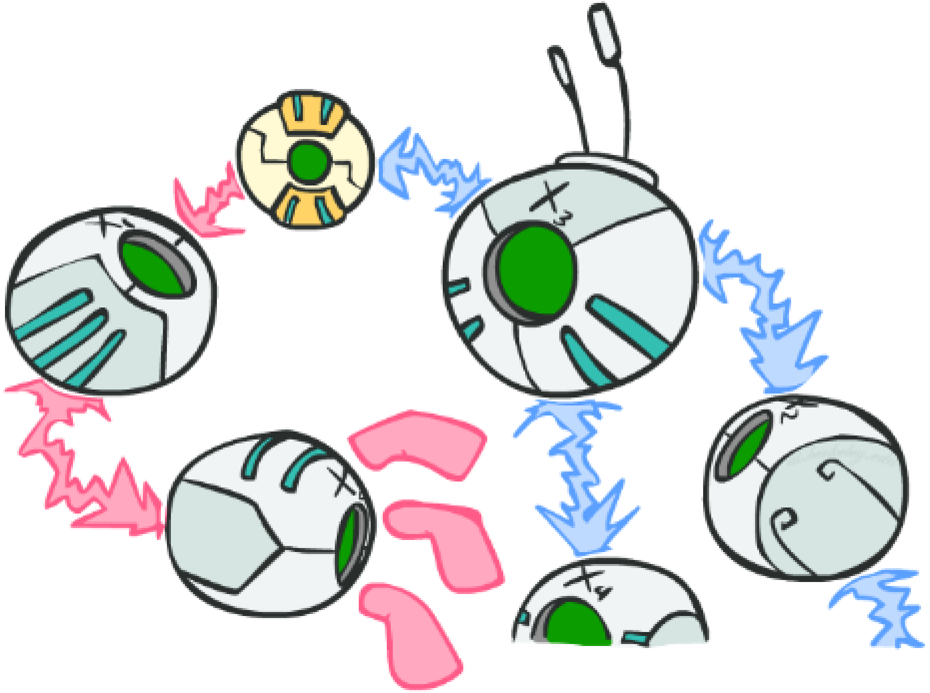
D-Separation

- Query: $X_i \perp\!\!\!\perp X_j | \{X_{k_1}, ..., X_{k_n}\}$?
- Check all (undirected!) paths between X_i and X_j
  - If one or more active, then independence not guaranteed

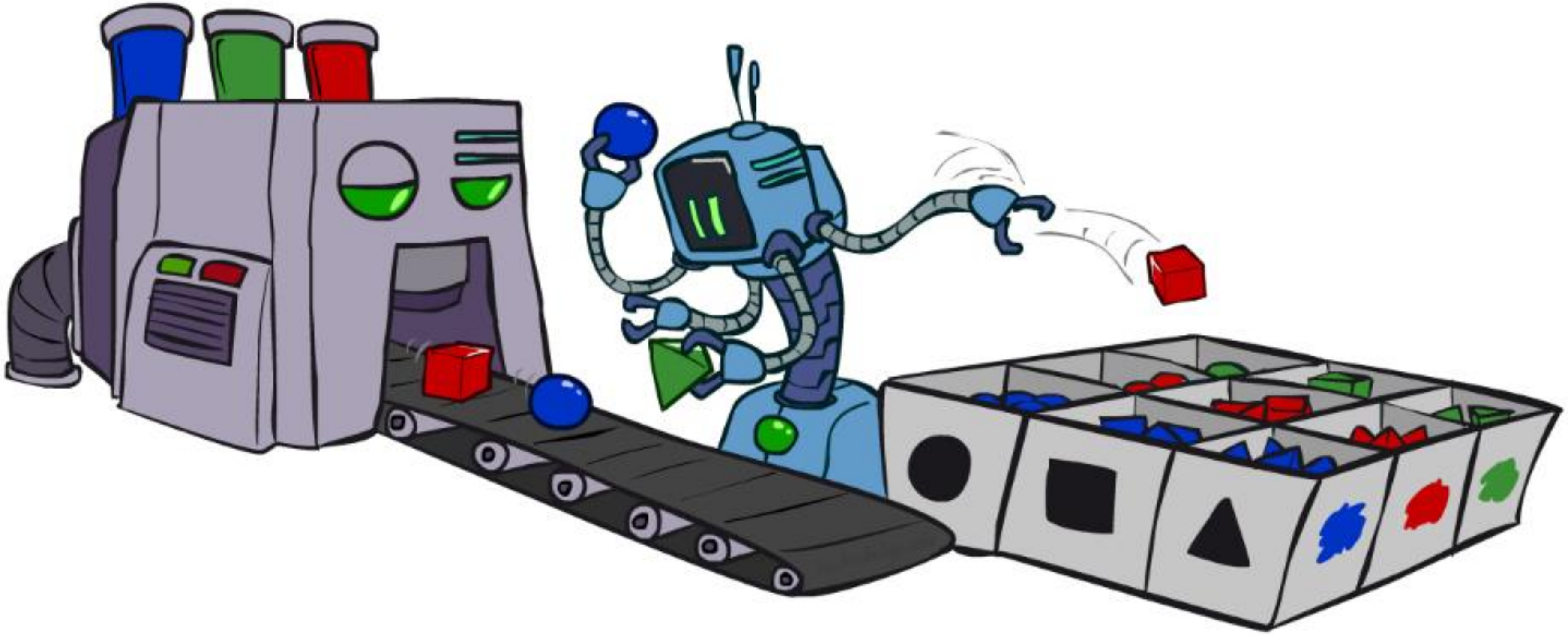
$$X_i \not\perp\!\!\!\perp X_j | \{X_{k_1}, ..., X_{k_n}\}$$

  - Otherwise (i.e. if all paths are inactive), then independence is guaranteed

$$X_i \perp\!\!\!\perp X_j | \{X_{k_1}, ..., X_{k_n}\}$$



Bayes' Nets: Sampling



Sampling

- Sampling from given distribution
  - Step 1: Get sample *u* from uniform distribution over [0, 1)

```
>>> import random
>>> random.random()
0.6303136415860905
```

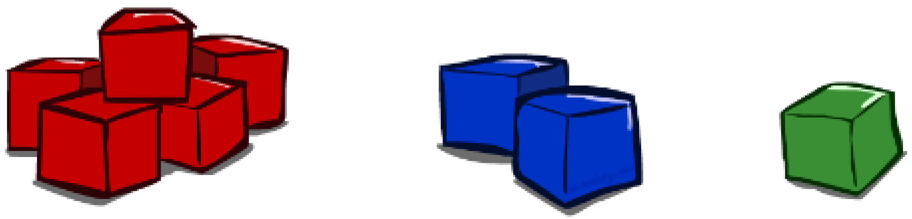
  - Step 2: Convert this sample *u* into an outcome for the given distribution by having each outcome associated with a sub-interval of [0,1) with sub-interval size equal to probability of the outcome

- Example

C	P(C)
red	0.6
green	0.1
blue	0.3

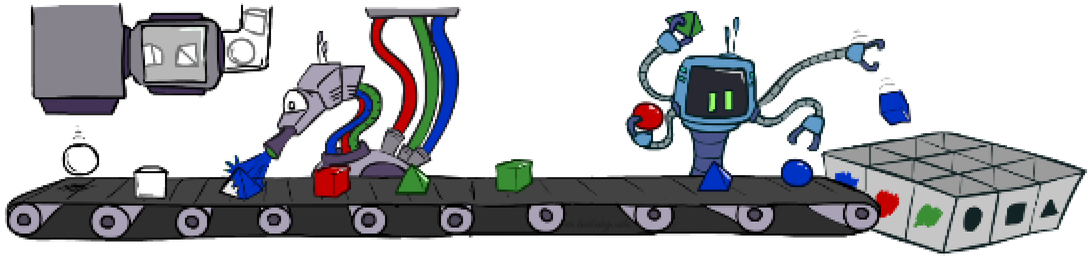
$$0 \leq u < 0.6, \rightarrow C = red$$
$$0.6 \leq u < 0.7, \rightarrow C = green$$
$$0.7 \leq u < 1, \rightarrow C = blue$$

  - If random() returns *u* = 0.83, then our sample is *C* = blue
  - E.g, after sampling 8 times:

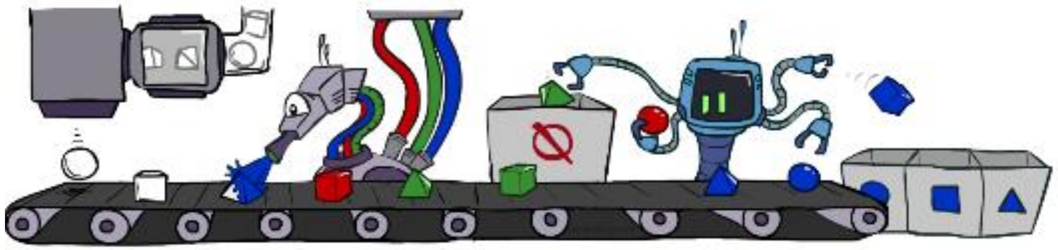


Bayes' Net Sampling Summary

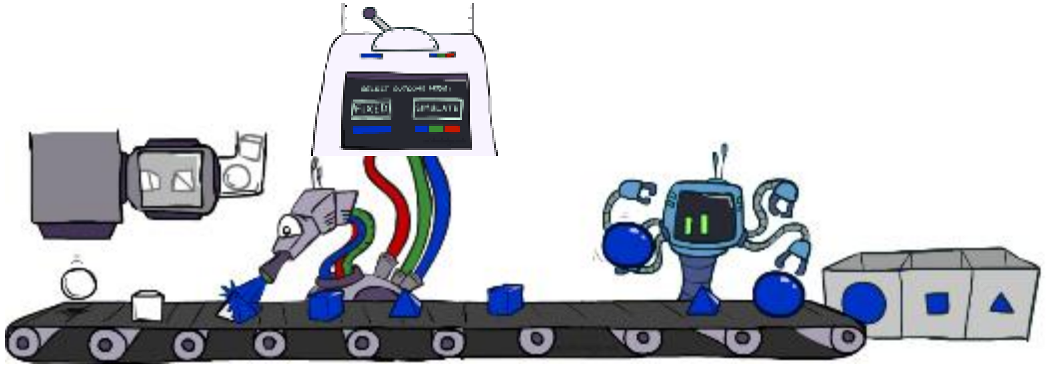
- Prior Sampling P



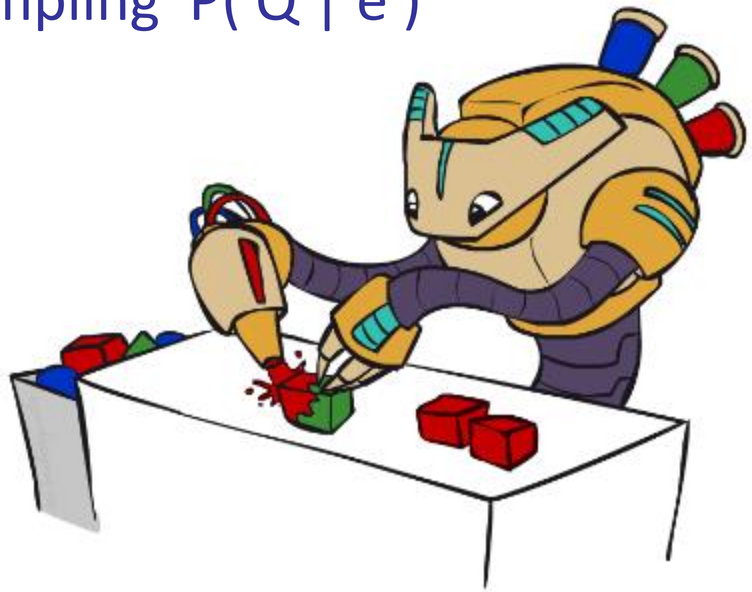
- Rejection Sampling P(Q | e)



- Likelihood Weighting P(Q | e)

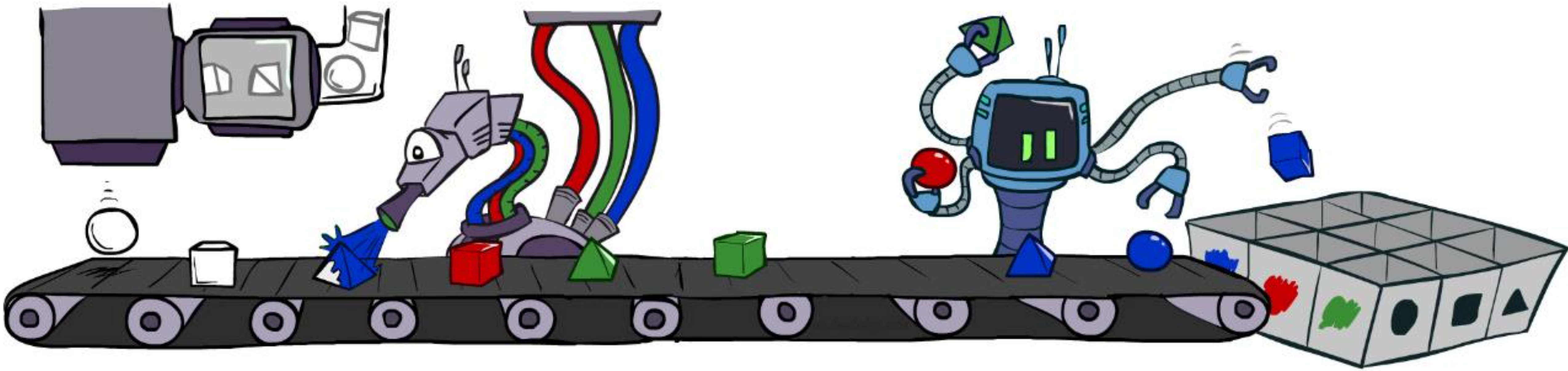


- Gibbs Sampling P(Q | e)



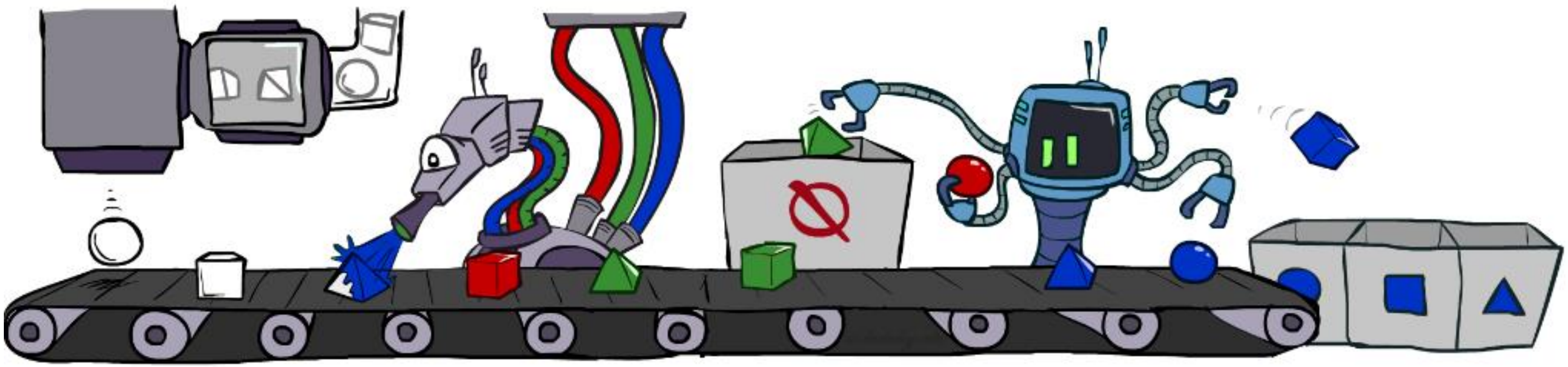
Prior Sampling

- For i=1, 2, …, n
  - Sample x_i from $P(X_i \mid Parents(X_i))$
- Return $(x_1, x_2, \ldots, x_n)$



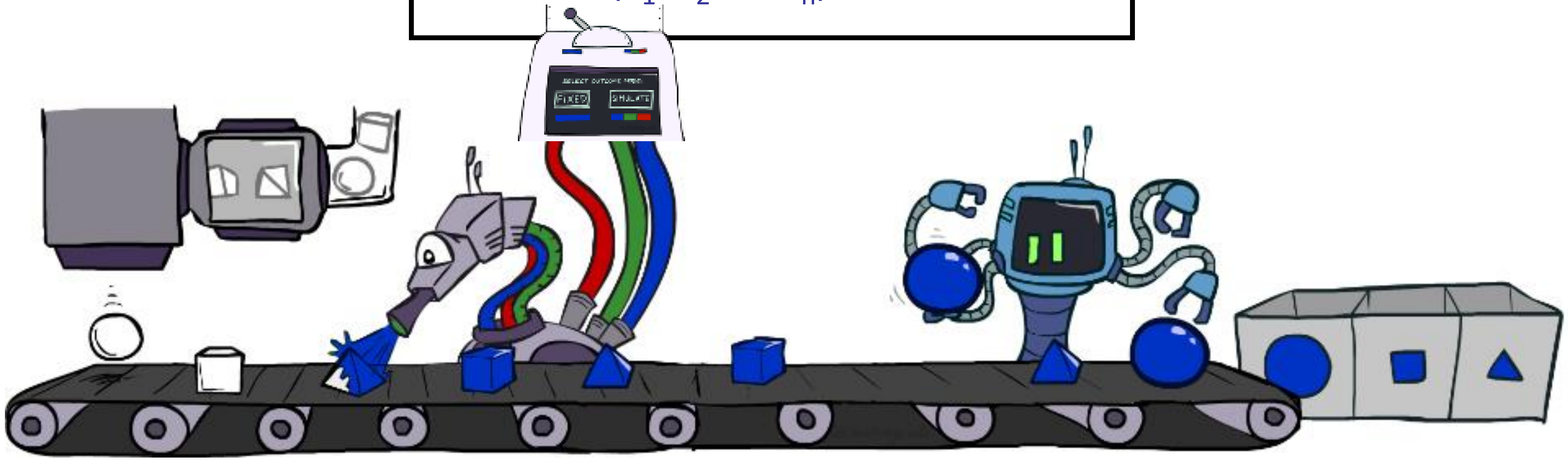
Rejection Sampling

- IN: evidence instantiation
- For i=1, 2, …, n
  - Sample x_i from $P(X_i \mid Parents(X_i))$
  - If x_i not consistent with evidence
    - Reject: Return, and no sample is generated in this cycle
- Return $(x_1, x_2, …, x_n)$



Likelihood Weighting

- IN: evidence instantiation
- w = 1.0
- for i=1, 2, …, n
  - if X_i is an evidence variable
    - X_i = observation x_i for X_i
    - Set w = w * $P(x_i \mid Parents(X_i))$
  - else
    - Sample x_i from $P(X_i \mid Parents(X_i))$
- return $(x_1, x_2, \ldots, x_n)$, w

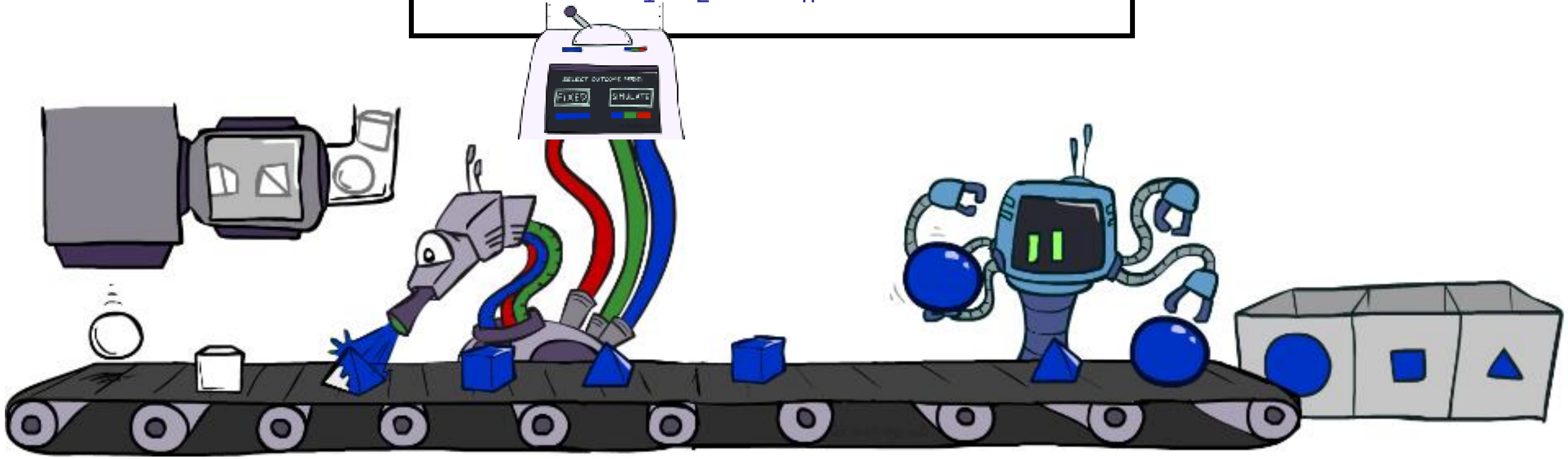


Likelihood Weighting

- IN: evidence instantiation
- w = 1.0
- for i=1, 2, …, n
  - if X_i is an evidence variable
    - X_i = observation x_i for X_i
    - Set w = w * P(x_i | Parents(X_i))
  - else
    - Sample x_i from P(X_i | Parents(X_i))
- return (x_1, x_2, …, x_n), w

Now each sample doesn't count as 1.0 but has a weight. Need to take a weighted average.

P(Q|Evidence) = Sum(weights of samples consistent with Query) / Total Weight of All samples.



Markov Models Recap

- Explicit assumption for all *t* : $X_t \perp\!\!\!\perp X_1, \ldots, X_{t-2} \mid X_{t-1}$
- Consequence, joint distribution can be written as:

$$P(X_1, X_2, \ldots, X_T) = P(X_1)P(X_2|X_1)P(X_3|X_2)\ldots P(X_T|X_{T-1})$$
$$= P(X_1)\prod_{t=2}^{T} P(X_t|X_{t-1})$$

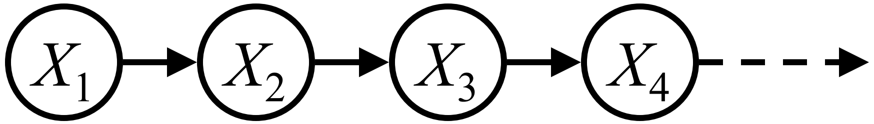
Huge savings in number of parameters needed!

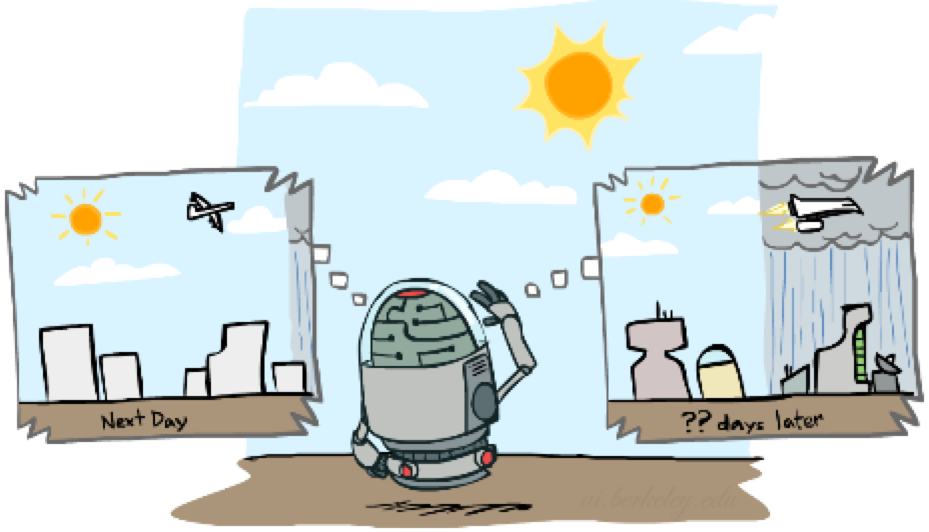
- Implied conditional independencies:
  - Past variables independent of future variables given the present

  i.e., if $t_1 < t_2 < t_3$ or $t_1 > t_2 > t_3$ then: $X_{t_1} \perp\!\!\!\perp X_{t_3} \mid X_{t_2}$
- Additional explicit assumption: $P(X_t \mid X_{t-1})$ is the same for all *t*

Mini-Forward Algorithm

- Question: What's P(X) on some day t?





$$P(x_1) = \text{known}$$

$$P(x_t) = \sum_{x_{t-1}} P(x_{t-1}, x_t)$$

$$= \sum_{x_{t-1}} P(x_t \mid x_{t-1}) P(x_{t-1})$$

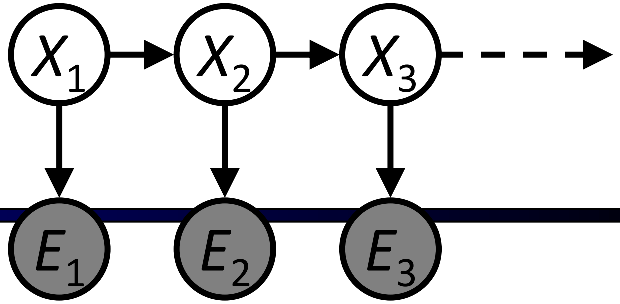
Forward simulation

Stationary Distributions

- For most chains:
  - Influence of the initial distribution gets less and less over time.
  - The distribution we end up in is independent of the initial distribution
- Stationary distribution:
  - The distribution we end up with is called the stationary distribution P_∞ of the chain
  - It satisfies

$$P_\infty(X) = P_{\infty+1}(X) = \sum_x P(X|x)P_\infty(x)$$

HMMs Recap



- Explicit assumption for all t : $X_t \perp\!\!\!\perp X_1, \ldots, X_{t-2} \mid X_{t-1}$
- Consequence, joint distribution can be written as:

$$P(X_1, X_2, \ldots, X_T) = P(X_1)P(X_2|X_1)P(X_3|X_2)\ldots P(X_T|X_{T-1})$$
$$= P(X_1)\prod_{t=2}^{T} P(X_t|X_{t-1})$$

- Implied conditional independencies:
  - Past variables independent of future variables given the present

i.e., if $t_1 < t_2 < t_3$ or $t_1 > t_2 > t_3$ then: $X_{t_1} \perp\!\!\!\perp X_{t_3} \mid X_{t_2}$

- Additional explicit assumption: $P(X_t \mid X_{t-1})$ is the same for all t

The Forward Algorithm

- We are given evidence at each time and want to know

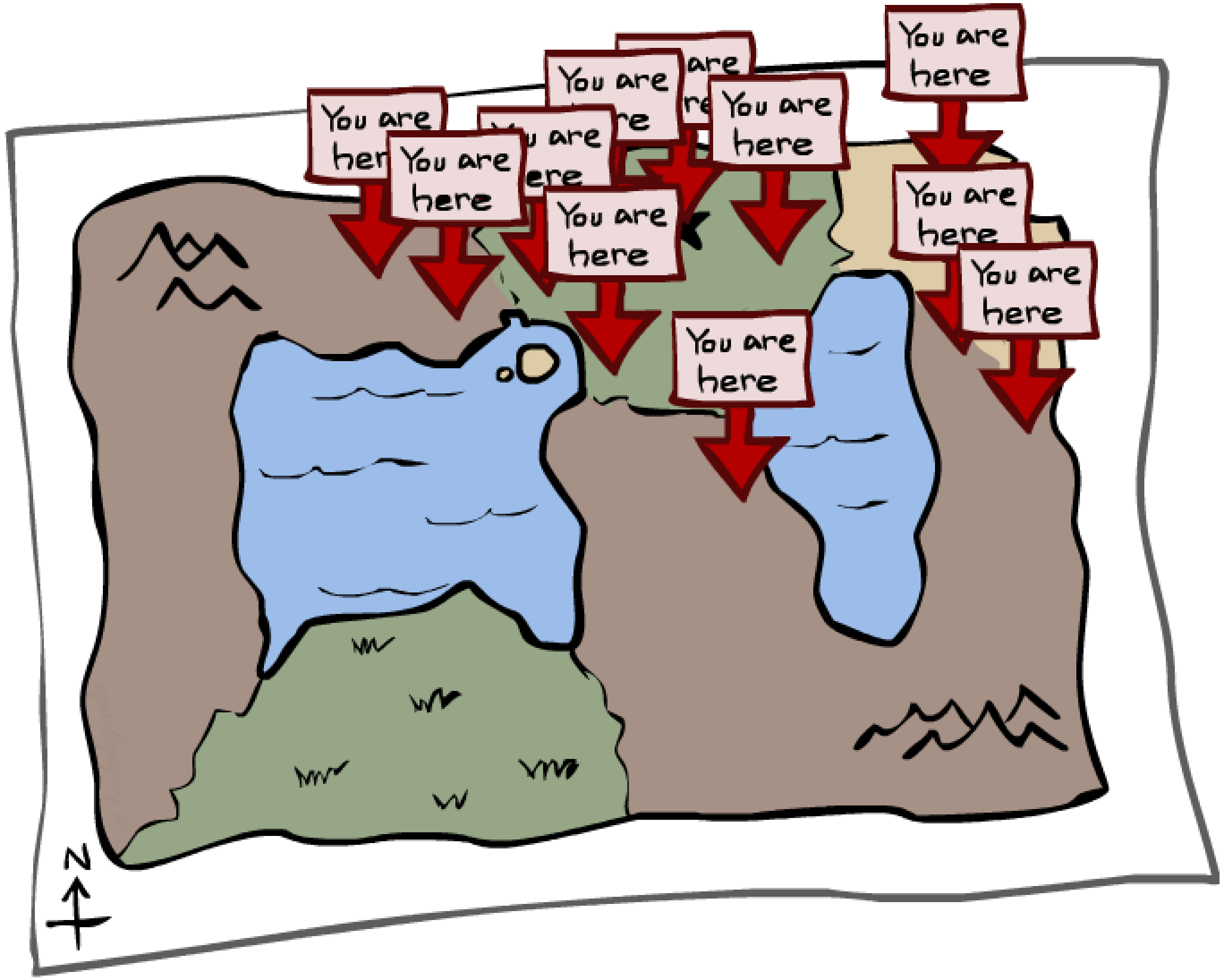
$$B_t(X) = P(X_t | e_{1:t})$$

- We can derive the following recursive update

$$P(X_t | e_{1:t}) = P(e_t | X_t) \sum_{x_{t-1}} P(X_t | X_{t-1}) P(X_{t-1} | e_{1:t-1})$$

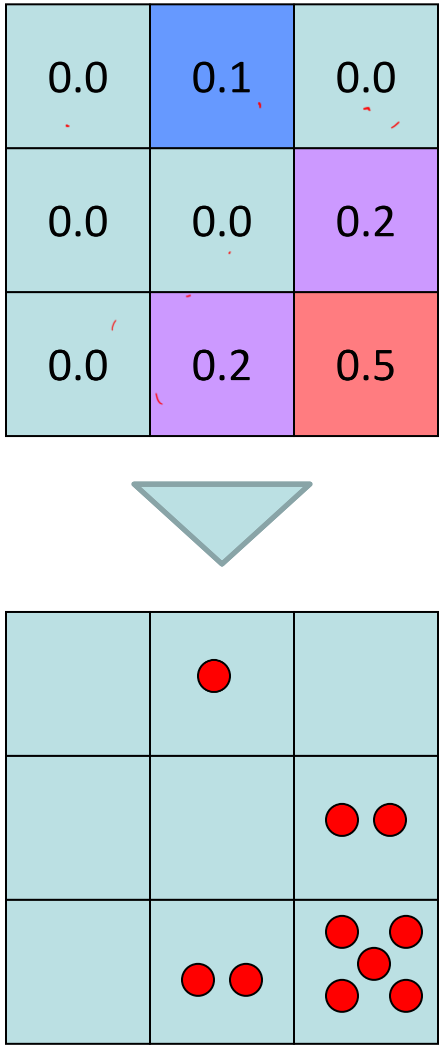
$$B_t(X) = P(e_t | X_t) \sum_{x_{t-1}} P(X_t | X_{t-1}) B_{t-1}(X)$$

Particle Filtering



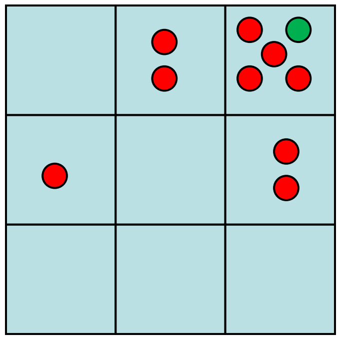
Particle Filtering

- Filtering: approximate solution
- Sometimes |X| is too big to use exact inference
  - |X| may be too big to even store B(X)
  - E.g. X is continuous
- Solution: approximate inference
  - Track samples of X, not all values
  - Samples are called particles
  - Time per step is linear in the number of samples
  - But: number needed may be large
  - In memory: list of particles, not states
- This is how robot localization works in practice
- Particle is just new name for sample



Representation: Particles

- Our representation of P(X) is now a list of N particles (samples)
  - Generally, N << |X|
  - Storing map from X to counts would defeat the point
- P(x) approximated by number of particles with value x
  - So, many x may have P(x) = 0!
  - More particles, more accuracy
- For now, all particles have a weight of 1



Particles:
(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(1,2)
(3,3)
(3,3)
(2,3)

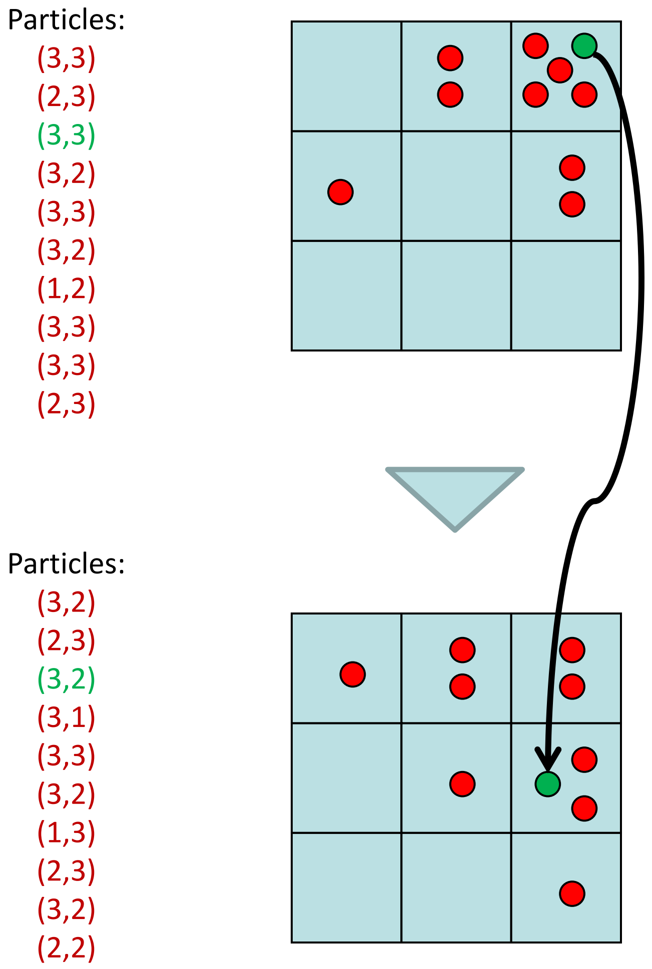
Particle Filtering: Elapse Time

- Each particle is moved by sampling its next position from the transition model

$$x' = \text{sample}(P(X'|x))$$

  - This is like prior sampling – samples' frequencies reflect the transition probabilities
  - Here, most samples move clockwise, but some move in another direction or stay in place

- This captures the passage of time
  - If enough samples, close to exact values before and after (consistent)



Particle Filtering: Observe

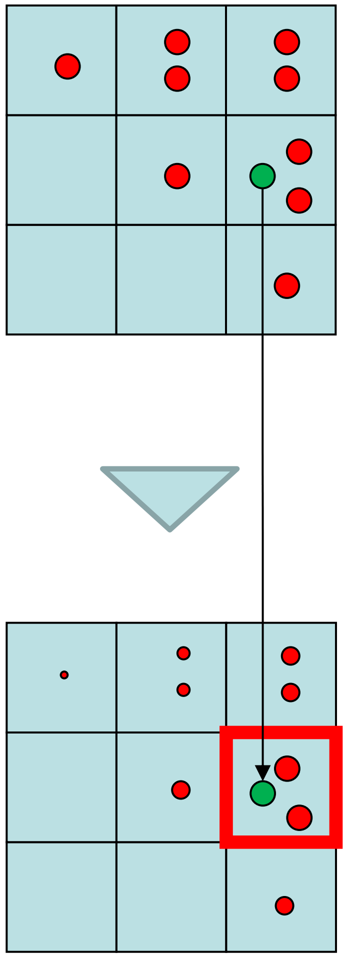
- Slightly trickier:
  - Don't sample observation, fix it
  - Similar to likelihood weighting, downweight samples based on the evidence

$$w(x) = P(e|x)$$

$$B(X) \propto P(e|X)B'(X)$$

  - As before, the probabilities don't sum to one, since all have been downweighted (in fact they now sum to (N times) an approximation of P(e))

Particles:
(3,2)
(2,3)
(3,2)
(3,1)
(3,3)
(3,2)
(1,3)
(2,3)
(3,2)
(2,2)

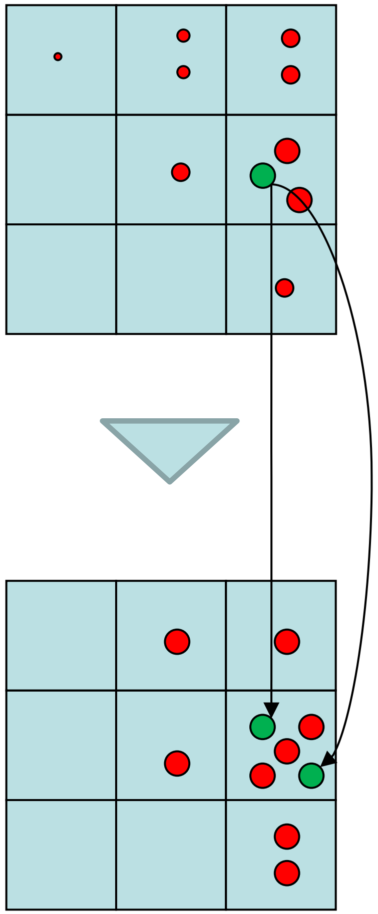


Particles:
(3,2) w=.9
(2,3) w=.2
(3,2) w=.9
(3,1) w=.4
(3,3) w=.4
(3,2) w=.9
(1,3) w=.1
(2,3) w=.2
(3,2) w=.9
(2,2) w=.4

Particle Filtering: Resample

- Rather than tracking weighted samples, we resample
- N times, we choose from our weighted sample distribution (i.e. draw with replacement)
- This is equivalent to renormalizing the distribution
- Now the update is complete for this time step, continue with the next one

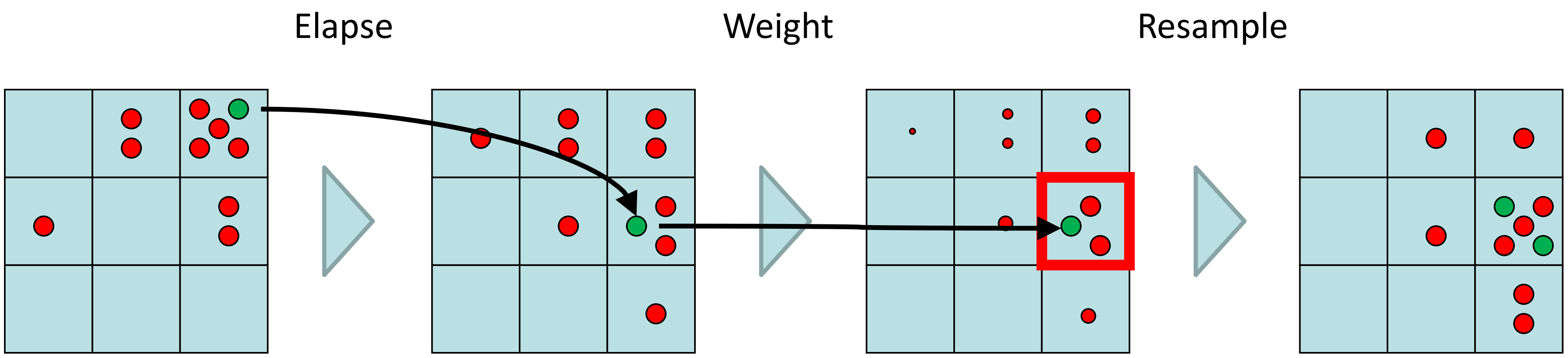
Particles:
(3,2) w=.9
(2,3) w=.2
(3,2) w=.9
(3,1) w=.4
(3,3) w=.4
(3,2) w=.9
(1,3) w=.1
(2,3) w=.2
(3,2) w=.9
(2,2) w=.4



(New) Particles:
(3,2)
(2,2)
(3,2)
(2,3)
(3,3)
(3,2)
(1,3)
(2,3)
(3,2)
(3,2)

Recap: Particle Filtering

- Particles: track samples of states rather than an explicit distribution



Particles:	Particles:	Particles:	(New) Particles:
(3,3)	(3,2)	(3,2) w=.9	(3,2)
(2,3)	(2,3)	(2,3) w=.2	(2,2)
(3,3)	(3,2)	(3,2) w=.9	(3,2)
(3,2)	(3,1)	(3,1) w=.4	(2,3)
(3,3)	(3,3)	(3,3) w=.4	(3,3)
(3,2)	(3,2)	(3,2) w=.9	(3,2)
(1,2)	(1,3)	(1,3) w=.1	(1,3)
(3,3)	(2,3)	(2,3) w=.2	(2,3)
(3,3)	(3,2)	(3,2) w=.9	(3,2)
(2,3)	(2,2)	(2,2) w=.4	(3,2)

[Demos: ghostbusters particle filtering (L15D3,4,5)]

Behavioral Cloning

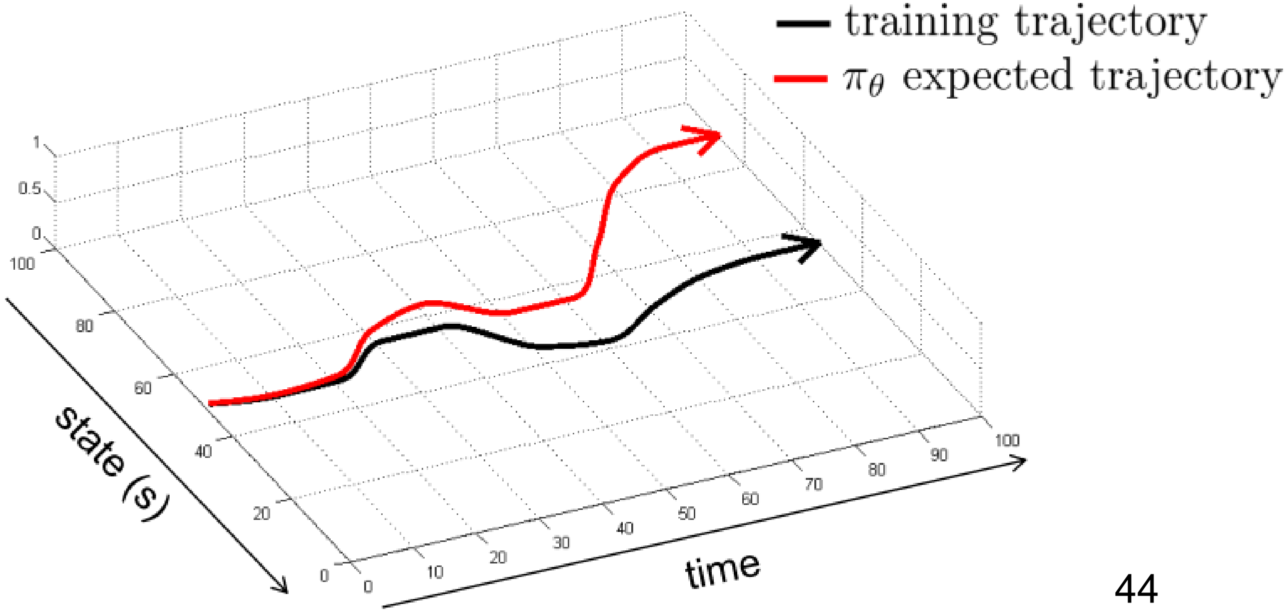
$\mathbf{o}_t$

$\mathbf{a}_t$

training data

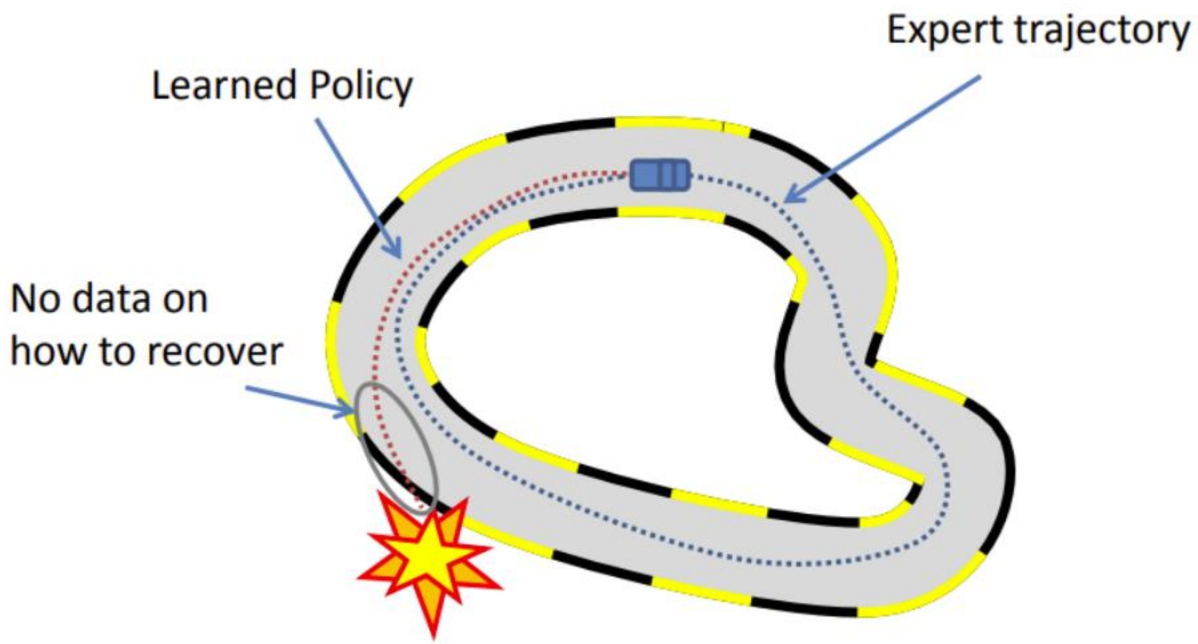
supervised learning

$\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$



Distribution Shift

$$p_{\pi^*}(o_t) \neq p_{\pi_\theta}(o_t)$$



	Supervised Learning	Supervised Learning + Control
Train	$(x, y) \sim D$	$s \sim P(\cdot \mid s, \pi^*(s))$
Test	$(x, y) \sim D$	$s \sim P(\cdot \mid s, \pi(s))$

DAgger

can we make $p_{\text{data}}(\mathbf{o}_t) = p_{\pi_\theta}(\mathbf{o}_t)$?

idea: instead of being clever about $p_{\pi_\theta}(\mathbf{o}_t)$, be clever about $p_{\text{data}}(\mathbf{o}_t)$!

DAgger: **D**ataset **Agg**regation

goal: collect training data from $p_{\pi_\theta}(\mathbf{o}_t)$ instead of $p_{\text{data}}(\mathbf{o}_t)$

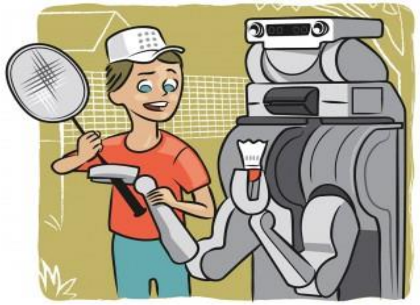
how? just run $\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$

but need labels $\mathbf{a}_t$!

1. train $\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$ from human data $\mathcal{D} = \{\mathbf{o}_1, \mathbf{a}_1, \ldots, \mathbf{o}_N, \mathbf{a}_N\}$
2. run $\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$ to get dataset $\mathcal{D}_\pi = \{\mathbf{o}_1, \ldots, \mathbf{o}_M\}$
3. Ask human to label $\mathcal{D}_\pi$ with actions $\mathbf{a}_t$
4. Aggregate: $\mathcal{D} \leftarrow \mathcal{D} \cup \mathcal{D}_\pi$

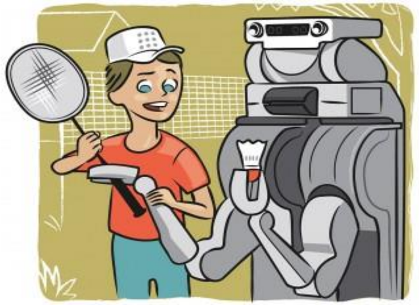
Ross et al. '11

Behavioral Cloning

 $\Rightarrow \pi$

- Answers the “How?” question
- Mimic the demonstrator
- Learn mapping from states to actions
- Computationally efficient
- Compounding errors

Inverse Reinforcement Learning

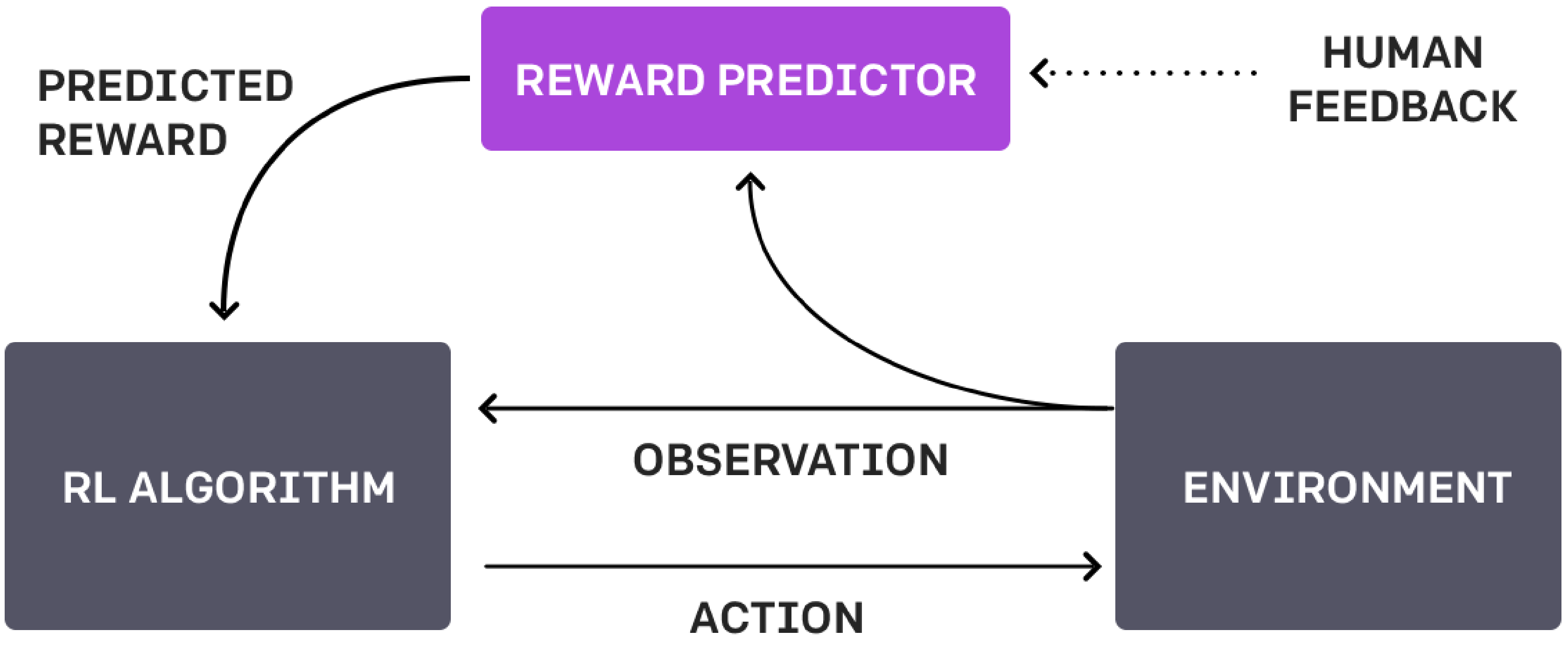
 $\Rightarrow R \Rightarrow \pi$

- Answers the “Why?” question
- Explain the demonstrator’s behavior
- Learn a reward function capturing the demonstrator’s intent
- Can require lots of data and compute
- Better generalization. Can recover from arbitrary states

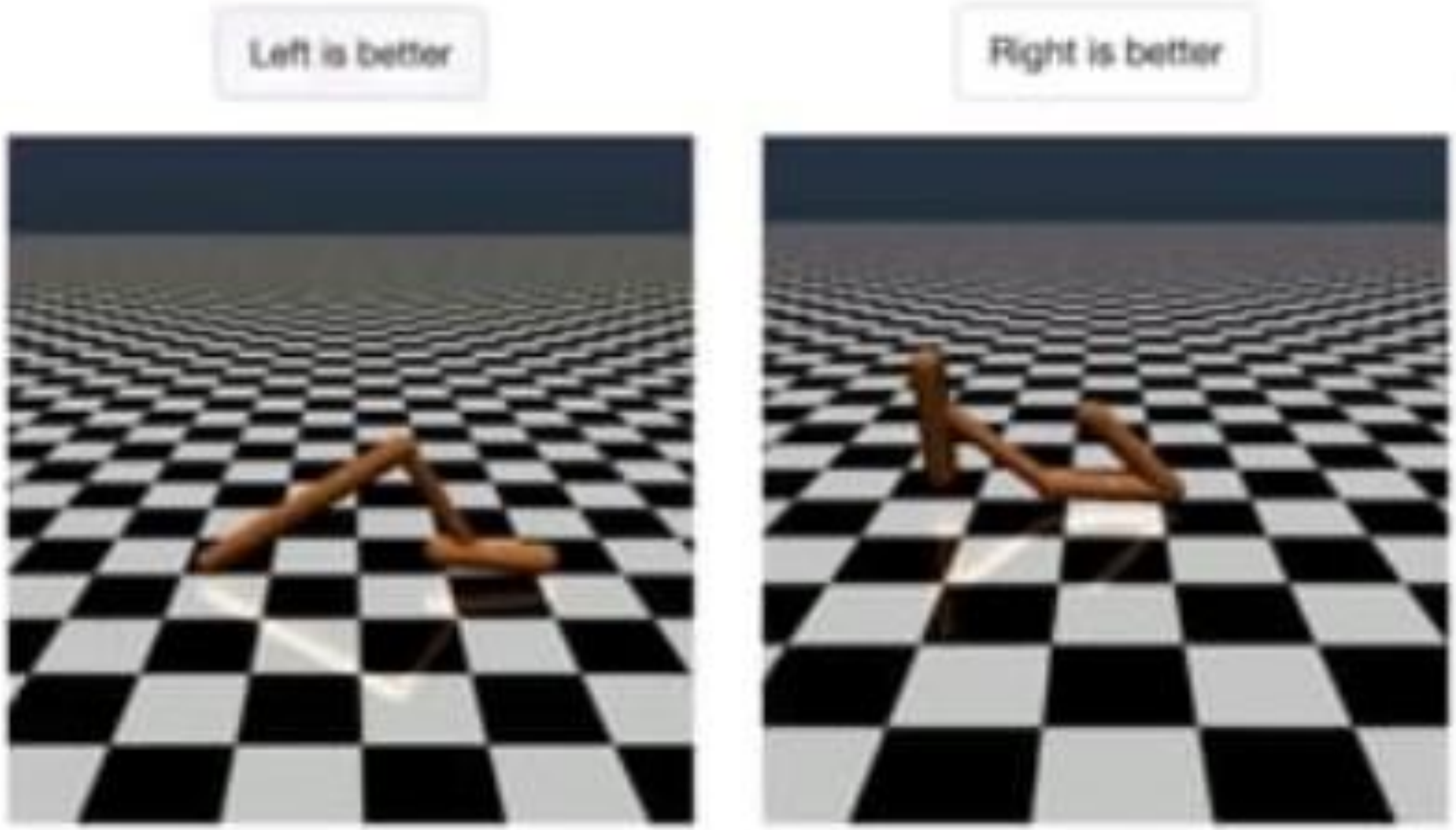
Basic IRL Algorithm

- Start with demonstrations, D
- Guess initial reward function R_0
- $\hat{R} = R_0$
- Loop:
  - Solve for optimal policy $\pi^*_{\hat{R}}$
  - Compare D and $\pi^*_{\hat{R}}$
  - Update $\hat{R}$ to try and make D and $\pi^*_{\hat{R}}$ more similar

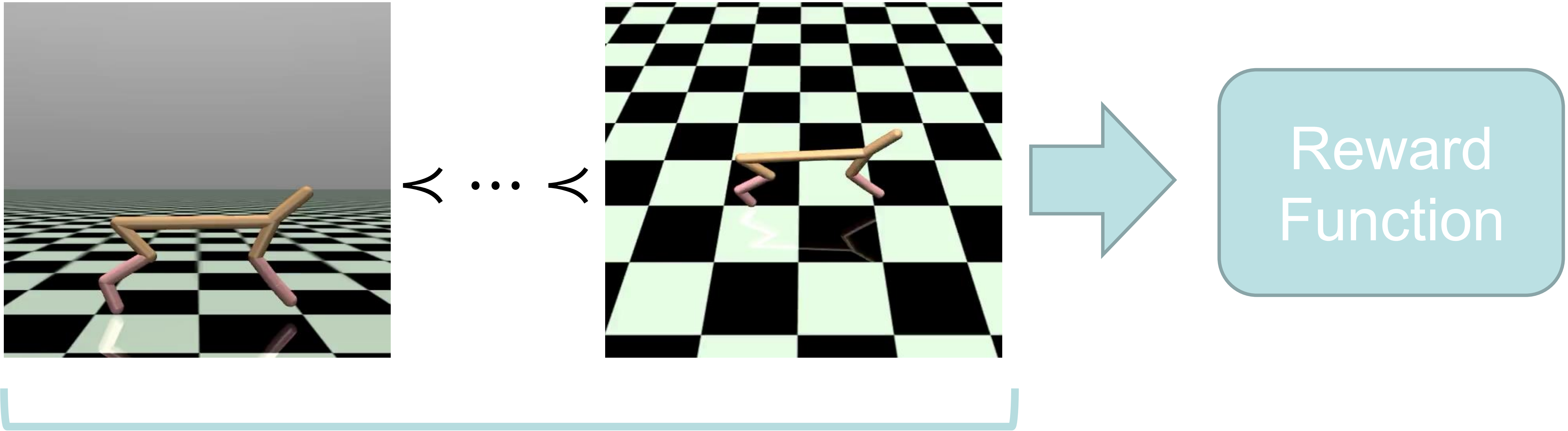
RL from Human Feedback (RLHF)



RL from Human Preferences



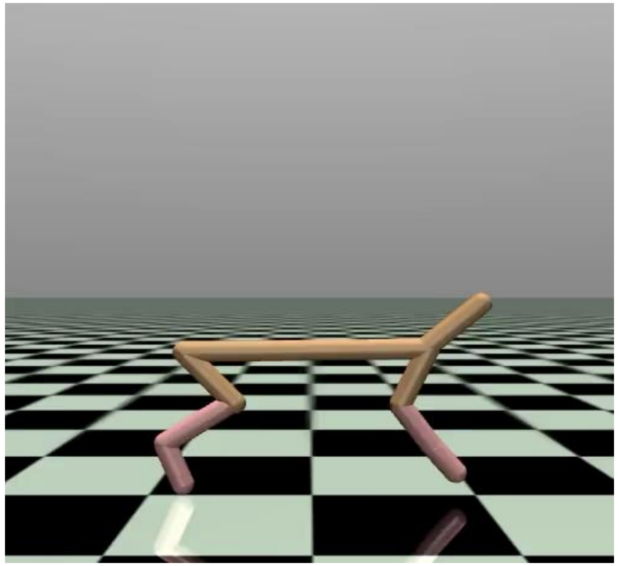
RLHF



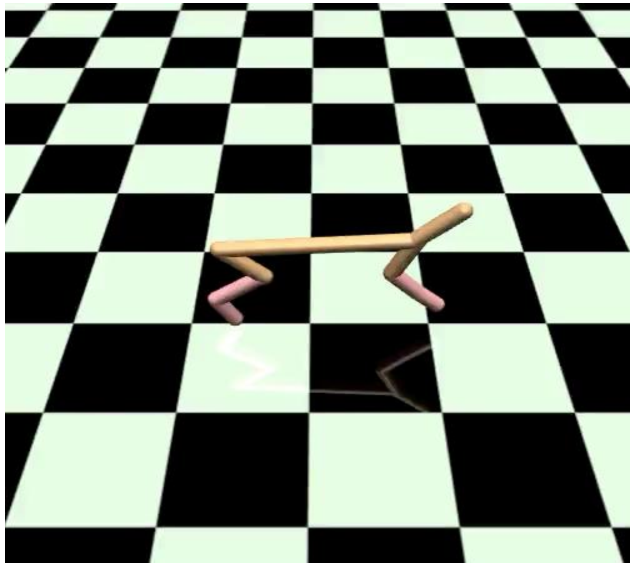
Pre-ranked demonstrations

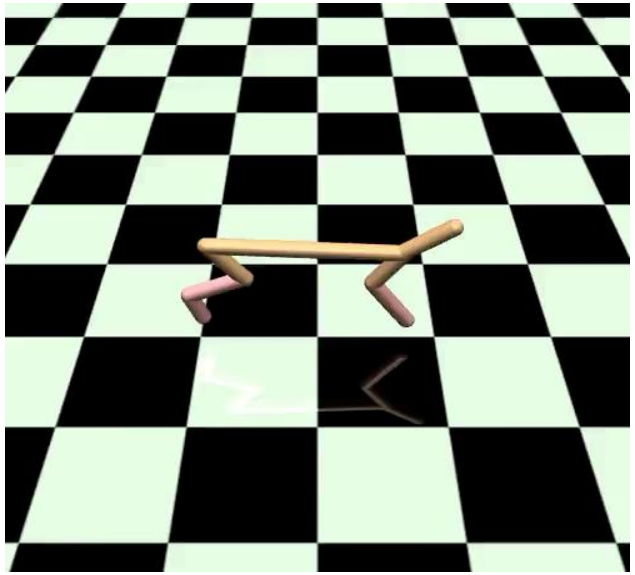
Brown et al. "Extrapolating Beyond Suboptimal Demonstrations via IRL from Observations." ICML 2019

RLHF



$\prec \cdots \prec$





Pre-ranked demonstrations

T-REX Policy

Brown et al. "Extrapolating Beyond Suboptimal Demonstrations via IRL from Observations." ICML 2019

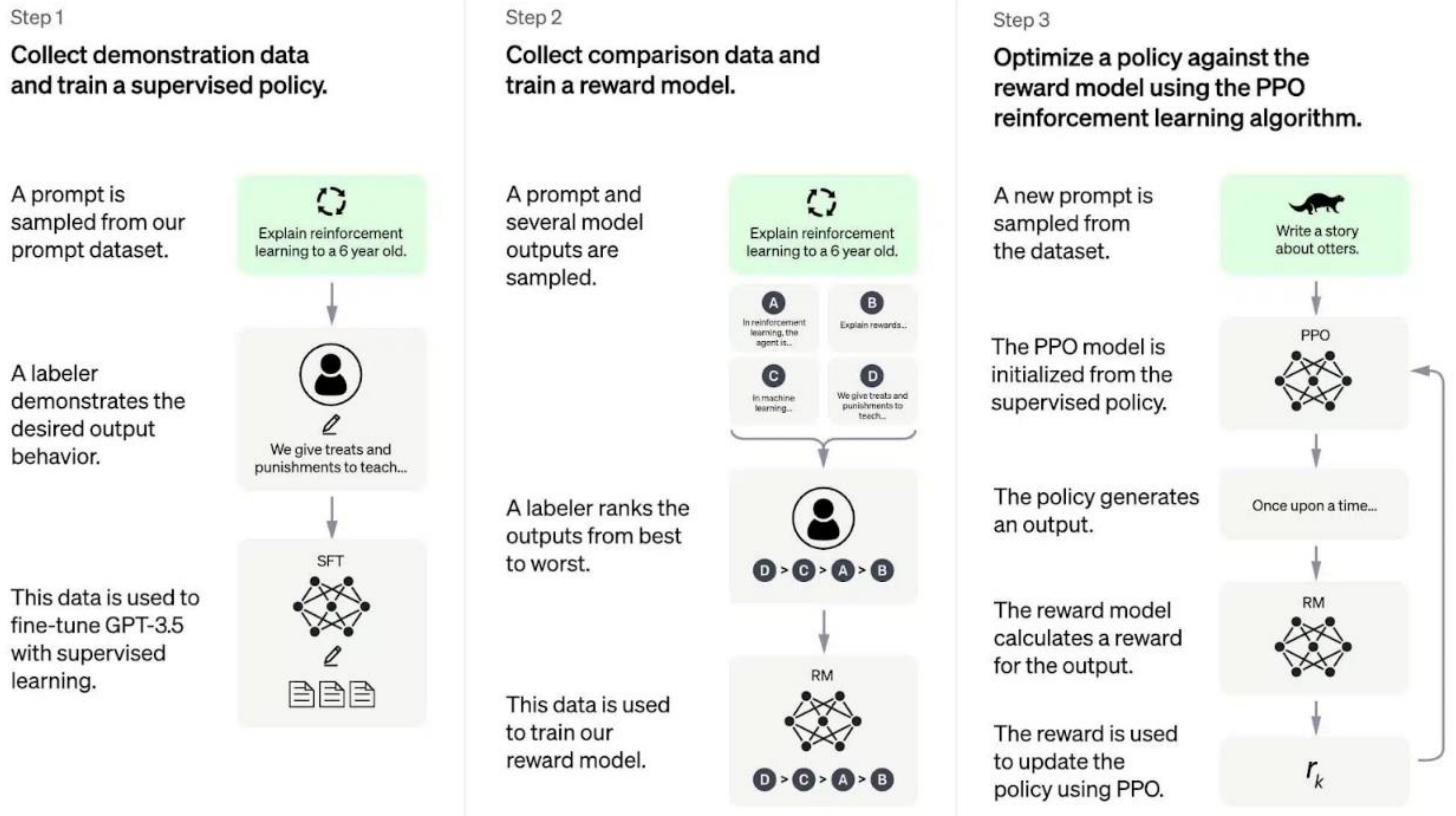
Learning from preferences

$$\tau_1 \prec \tau_2 \prec \cdots \prec \tau_T$$

$$\sum_{s \in \tau_1} R_\theta(s) < \sum_{s \in \tau_2} R_\theta(s)$$

Bradley-Terry pairwise ranking loss

$$\mathcal{L}(\theta) = -\sum_{\tau_i \prec \tau_j} \frac{\exp \sum_{s \in \tau_j} R_\theta(s)}{\exp \sum_{s \in \tau_i} R_\theta(s) + \exp \sum_{s \in \tau_j} R_\theta(s)}$$


Step 1
Collect demonstration data and train a supervised policy.
A prompt is sampled from our prompt dataset.
Explain reinforcement learning to a 6 year old.
A labeler demonstrates the desired output behavior.
We give treats and punishments to teach...
This data is used to fine-tune GPT-3.5 with supervised learning.
SFT
Step 2
Collect comparison data and train a reward model.
A prompt and several model outputs are sampled.
Explain reinforcement learning to a 6 year old.
A
In reinforcement learning, the agent is...
B
Explain rewards...
C
In machine learning...
D
We give treats and punishments to teach...
A labeler ranks the outputs from best to worst.
D > C > A > B
This data is used to train our reward model.
RM
D > C > A > B
Step 3
Optimize a policy against the reward model using the PPO reinforcement learning algorithm.
A new prompt is sampled from the dataset.
Write a story about otters.
The PPO model is initialized from the supervised policy.
PPO
The policy generates an output.
Once upon a time...
The reward model calculates a reward for the output.
RM
The reward is used to update the policy using PPO.
r_k

We made it!