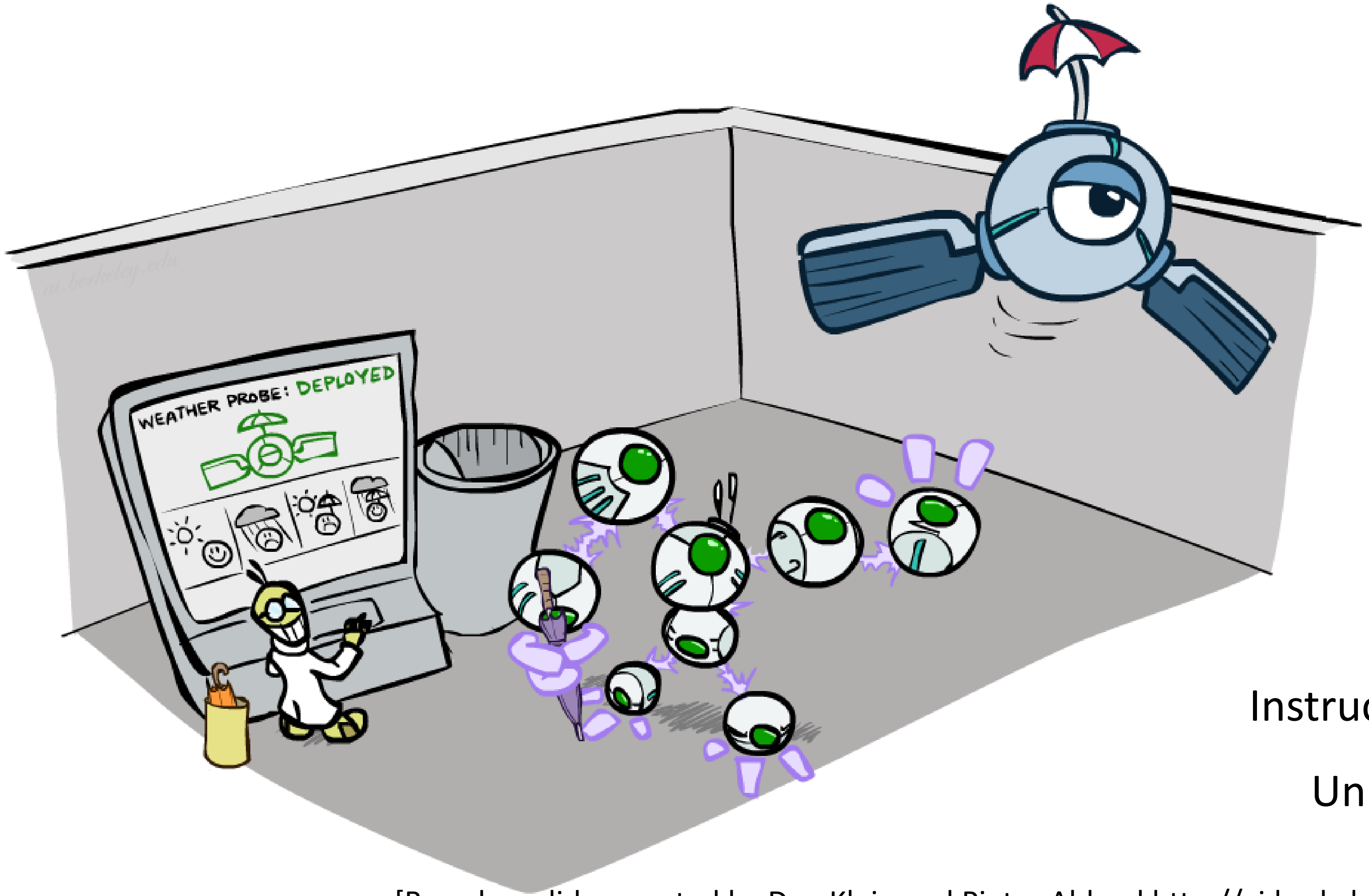
CS 6300: Artificial Intelligence

Decision Networks and Value of Perfect Information

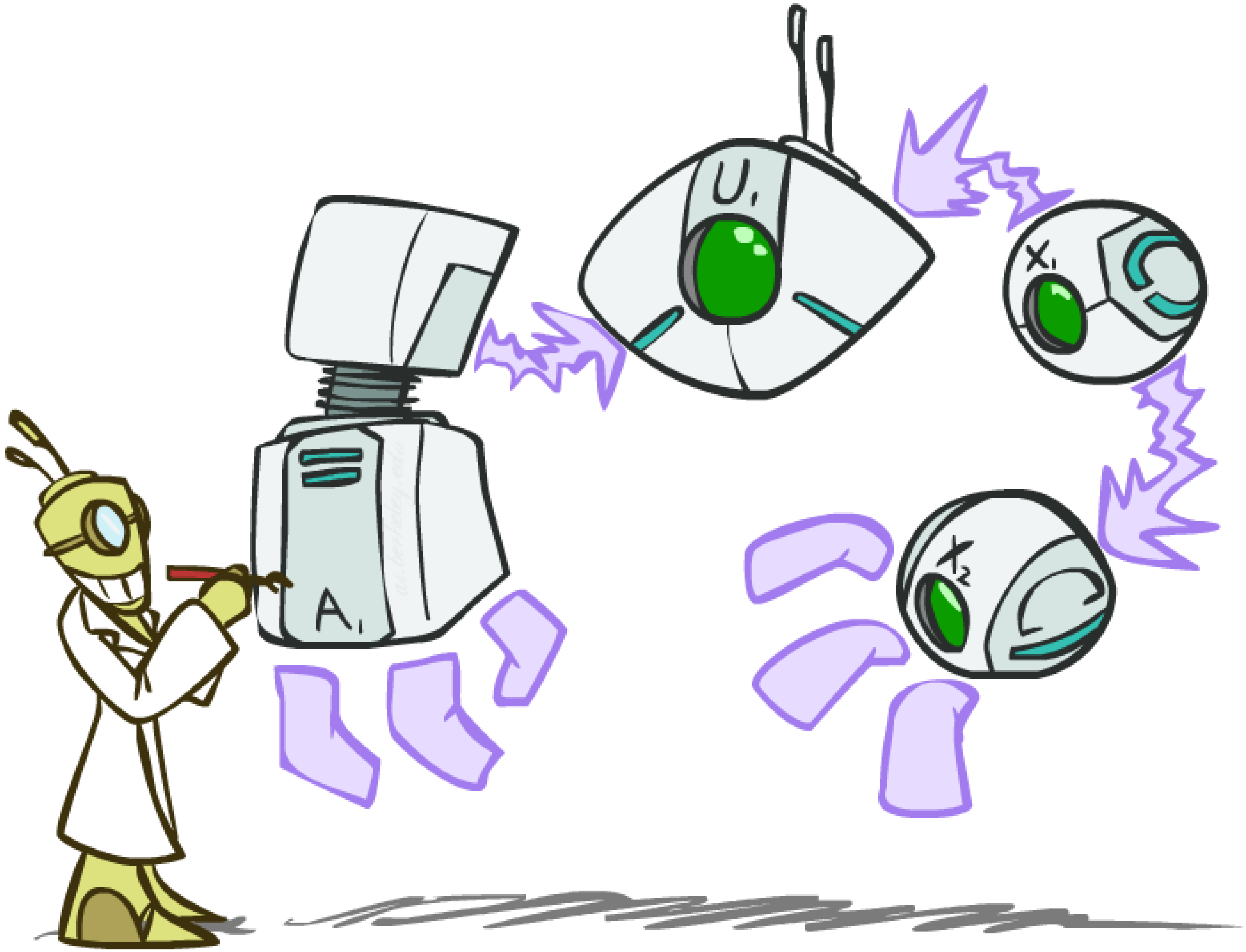


Instructor: Daniel Brown

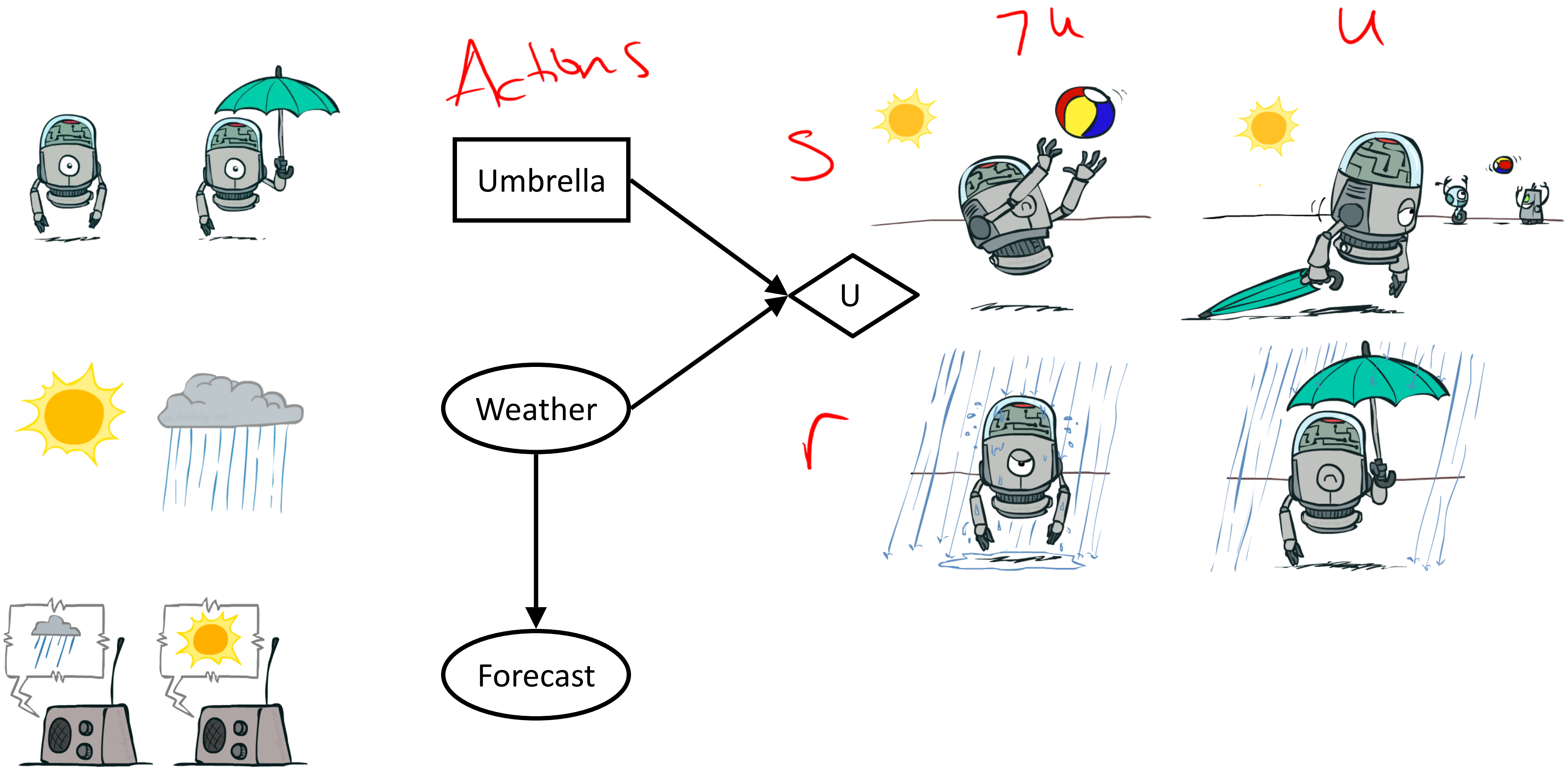
University of Utah

[Based on slides created by Dan Klein and Pieter Abbeel http://ai.berkeley.edu.]

Decision Networks

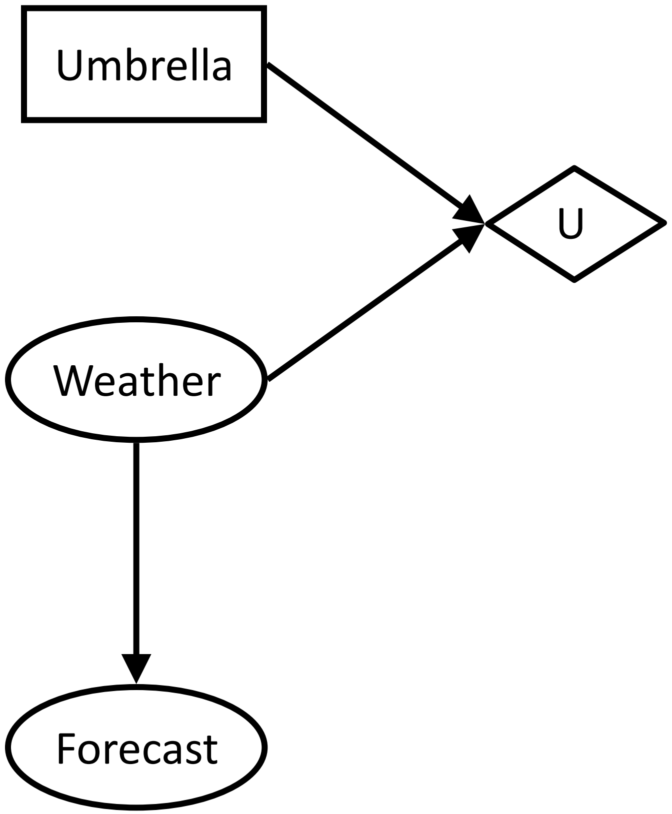


Decision Networks



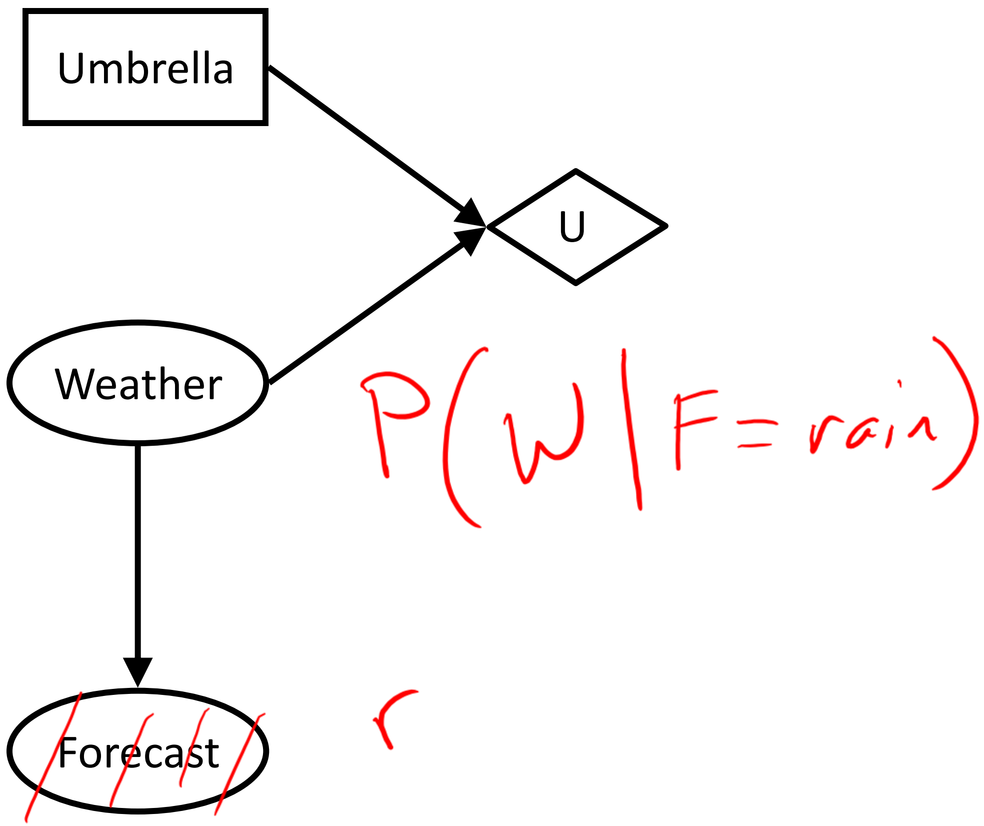
Decision Networks

- **Maximize Expected Utility**
  - **choose the action which maximizes the expected utility given the evidence**
- Can directly operationalize this with decision networks
  - Bayes nets with nodes for utility and actions
  - Lets us calculate the expected utility for each action
- New node types:
  - Chance nodes (just like BNs)
  - Actions (rectangles, cannot have parents, act as observed evidence)
  - Utility node (diamond, depends on action and chance nodes)



Decision Networks

- Action selection
  - Instantiate all evidence
  - Set action node(s) each possible way
  - Calculate posterior for all parents of utility node, given the evidence
  - Calculate expected utility for each action
  - Choose maximizing action



Decision Networks

Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}) = \sum_{w} P(w)U(\mathrm{leave}, w)$$

$$= 0.7 \cdot 100 + 0.3 \cdot 0 = 70$$

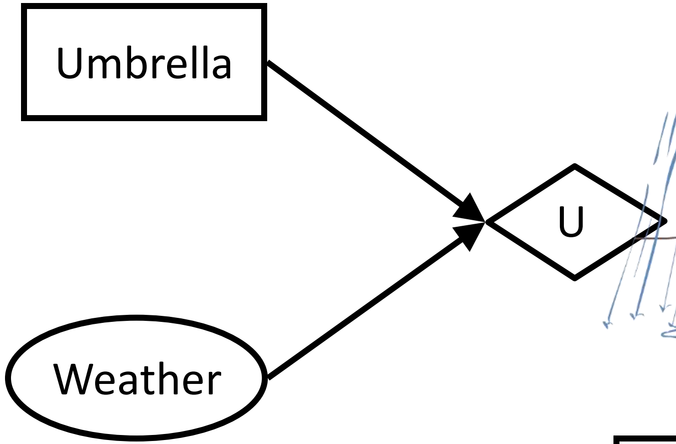
Umbrella = take

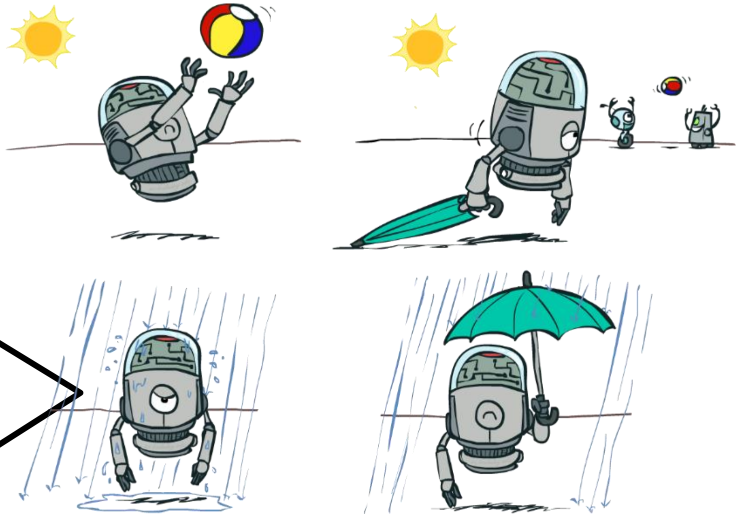
$$\mathrm{EU}(\mathrm{take}) = \sum_{w} P(w)U(\mathrm{take}, w)$$

$$= 0.7 \cdot 20 + 0.3 \cdot 70 = 35$$

Optimal decision = leave

$$\mathrm{MEU}(\emptyset) = \max_{a} \mathrm{EU}(a) = 70$$





W	P(W)
sun	0.7
rain	0.3

A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Example: Decision Networks

Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}|\mathrm{bad}) = \sum_w P(w|\mathrm{bad})U(\mathrm{leave}, w)$$

$$= 0.34 \cdot 100 + 0.66 \cdot 0 = 34$$

Umbrella = take

$$\mathrm{EU}(\mathrm{take}|\mathrm{bad}) = \sum_w P(w|\mathrm{bad})U(\mathrm{take}, w)$$

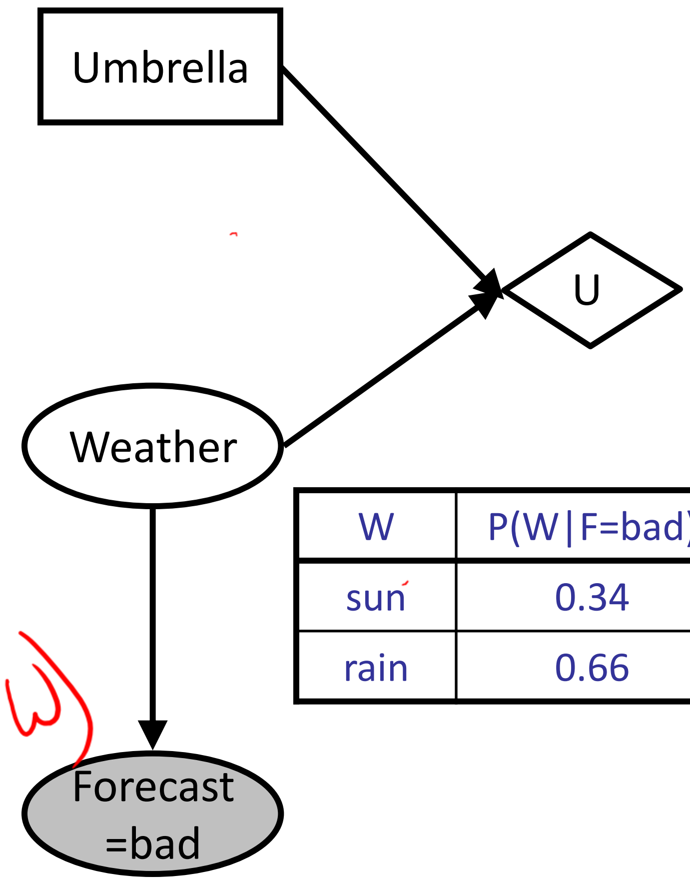
$$= 0.34 \cdot 20 + 0.66 \cdot 70 = 53$$

Optimal decision = take

$$\mathrm{MEU}(F = \mathrm{bad}) = \max_a \mathrm{EU}(a|\mathrm{bad}) = 53$$

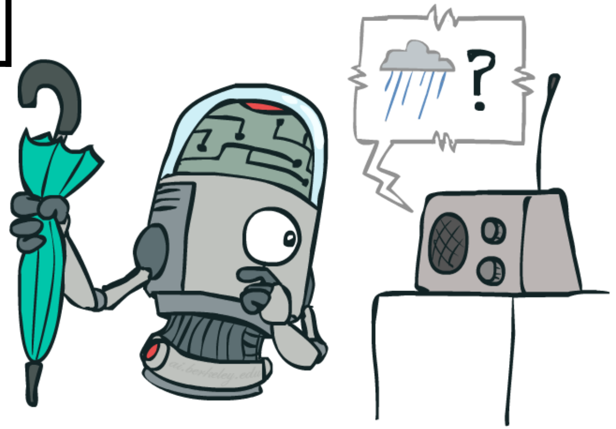
P(W)

P(F/W)

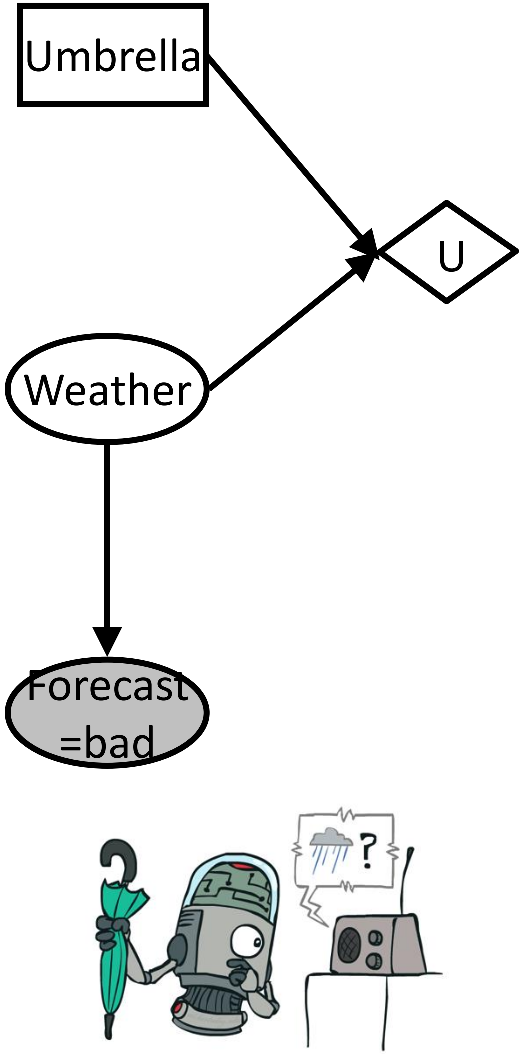


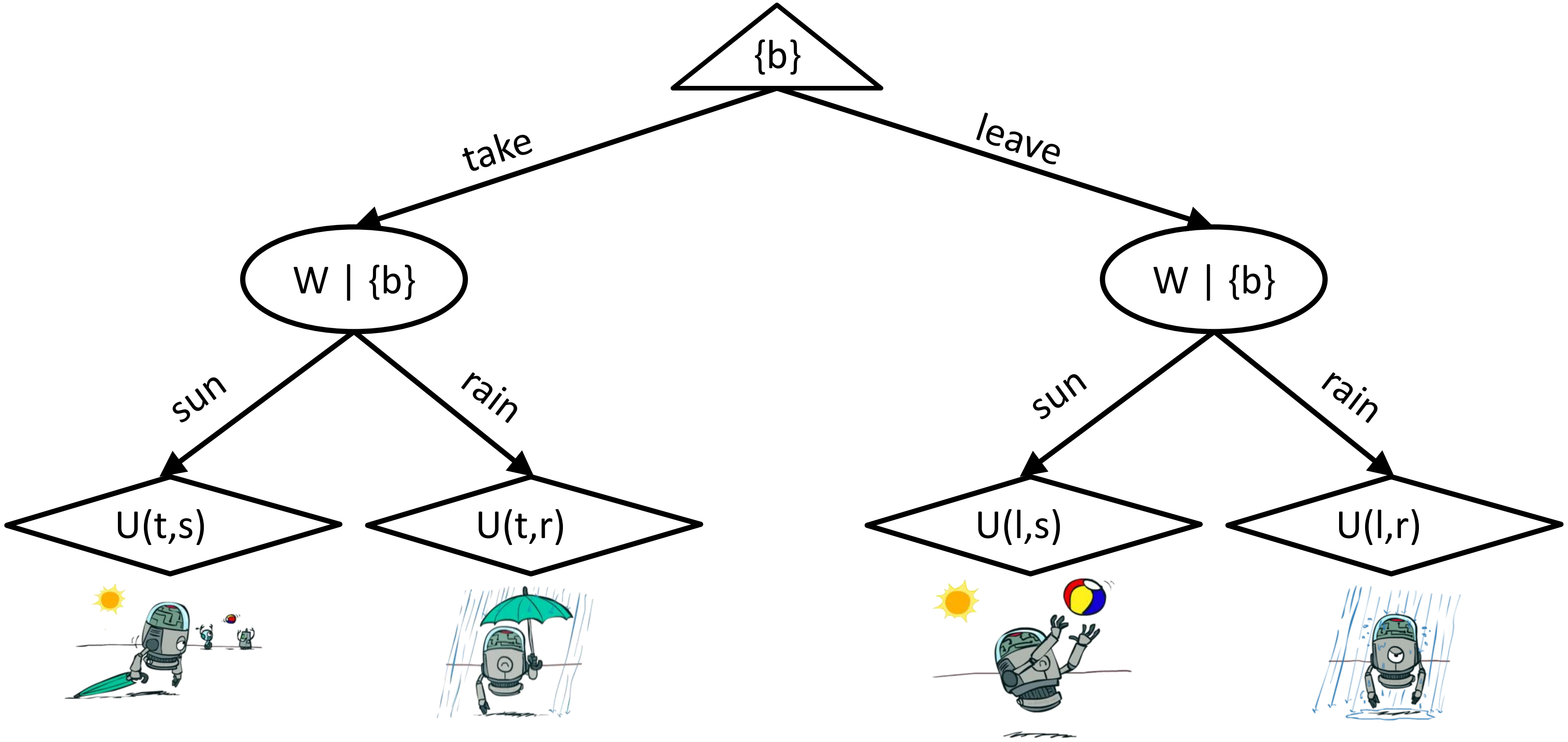
A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

W	P(W|F=bad)
sun	0.34
rain	0.66



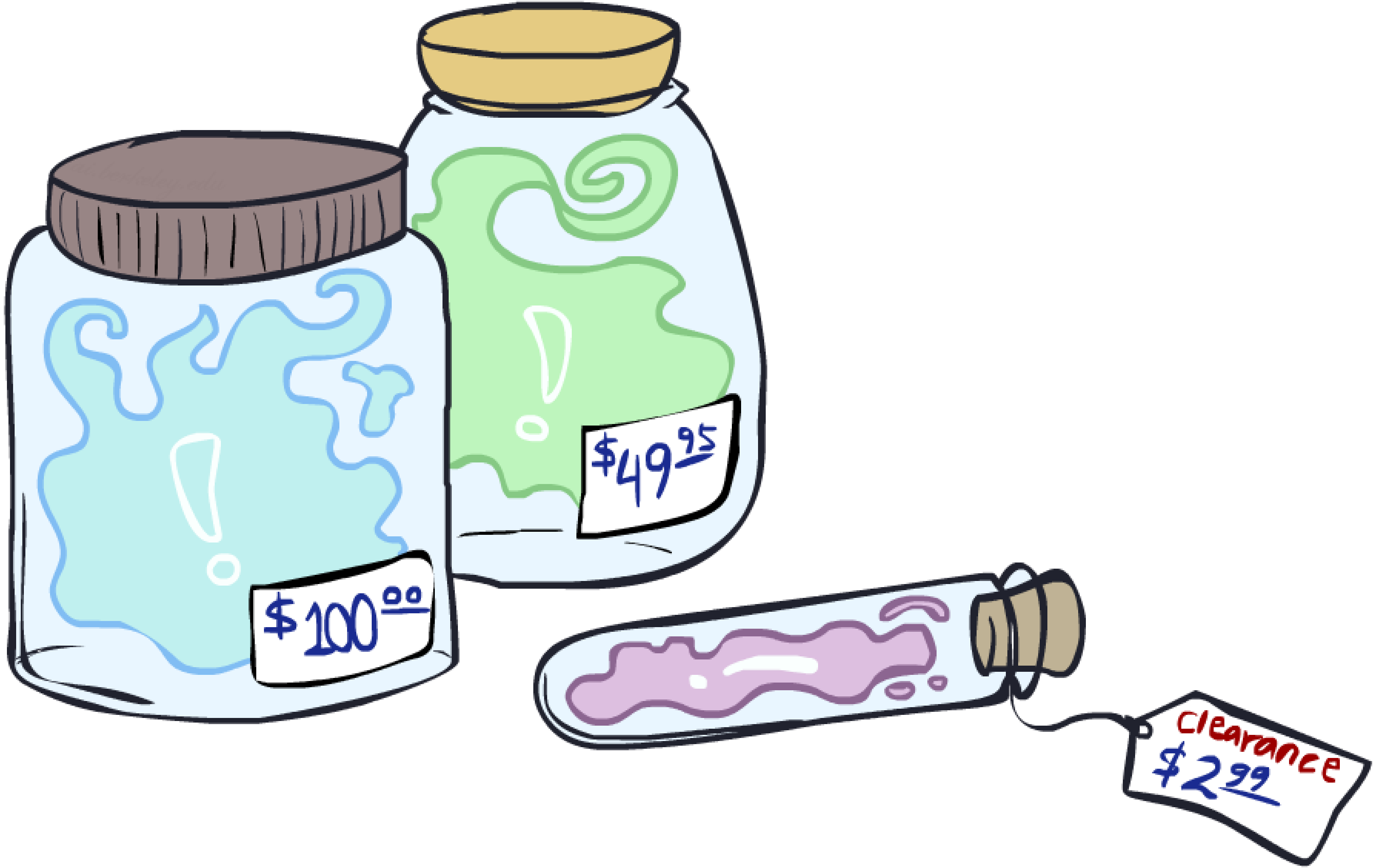
Decisions as Outcome Trees





Decision diagram tells us what numbers to put in our expectimax tree!

Value of Information

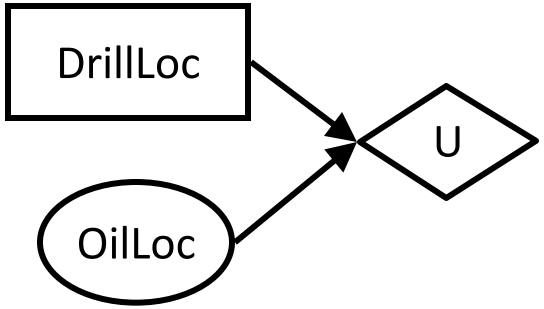


Value of Perfect Information (VPI)

- Idea: compute value of acquiring evidence
  - Can be done directly from decision network
- Example: buying oil drilling rights
  - Two blocks A and B, exactly one has oil, worth k
  - You can drill in one location
  - Prior probabilities 0.5 each, & mutually exclusive
  - Drilling in either A or B has EU = k/2, MEU = k/2
- Question: what' s the value of information of O?
  - Value of knowing which of A or B has oil
  - Value is expected gain in MEU from new info
  - Survey may say "oil in a" or "oil in b," prob 0.5 each
  - If we know OilLoc, MEU is k (either way)
  - Gain in MEU from knowing OilLoc?
  - VPI(OilLoc) = k/2
  - Fair price of information: k/2

$k - \frac{k}{2} = \frac{k}{2}$

action

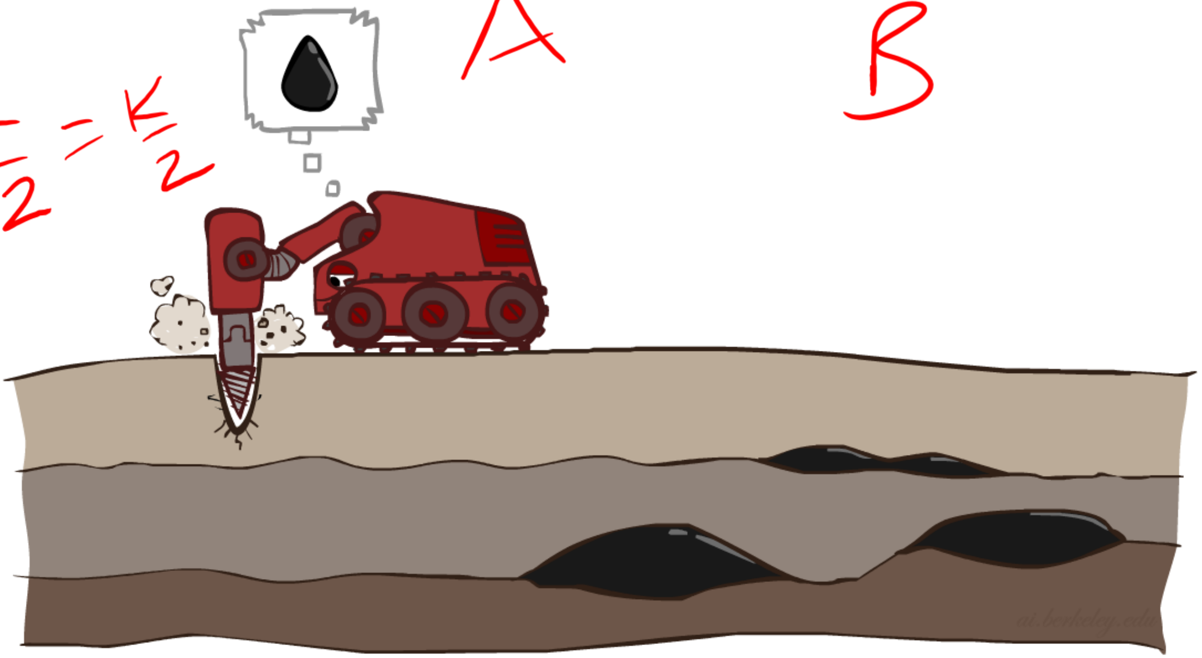


O	P
a	1/2
b	1/2

D	O	U
a	a	k
a	b	0
b	a	0
b	b	k

A

B



VPI Example: Weather

MEU with no evidence

$$\mathrm{MEU}(\phi) = \max_a \mathrm{EU}(a) = 70$$

MEU if forecast is bad

$$\mathrm{MEU}(F=\mathrm{bad}) = \max_a \mathrm{EU}(a|\mathrm{bad}) = 53$$

MEU if forecast is good

$$\mathrm{MEU}(F=\mathrm{good}) = \max_a \mathrm{EU}(a|\mathrm{good}) = 95$$

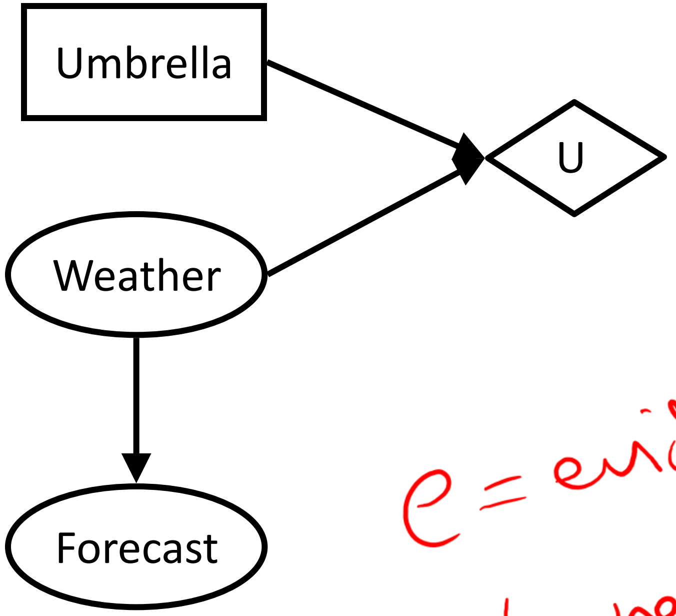
Forecast distribution

F	P(F)
good	0.59
bad	0.41

$$0.59 \cdot (95) + 0.41 \cdot (53) - 70$$

$$77.8 - 70 = 7.8$$

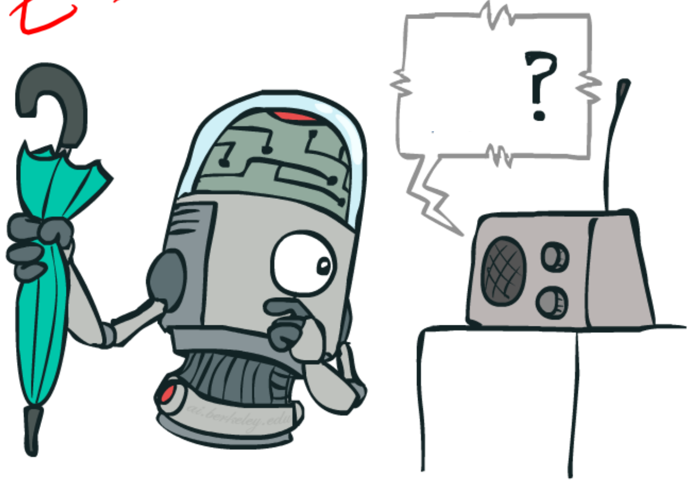
$$\mathrm{VPI}(E'|e) = \left(\sum_{e'} P(e'|e)\mathrm{MEU}(e,e')\right) - \mathrm{MEU}(e)$$



A	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

e = evidence we already have

E' = new evidence



Value of Information

A)

- Assume we have evidence E=e. Value if we act now:

$$\text{MEU}(e) = \max_a \sum_s P(s|e)\, U(s,a)$$

- Assume we see that E' = e'. Value if we act then:

$$\text{MEU}(e,e') = \max_a \sum_s P(s|e,e')\, U(s,a)$$

- BUT E' is a random variable whose value is unknown, so we don't know what e' will be

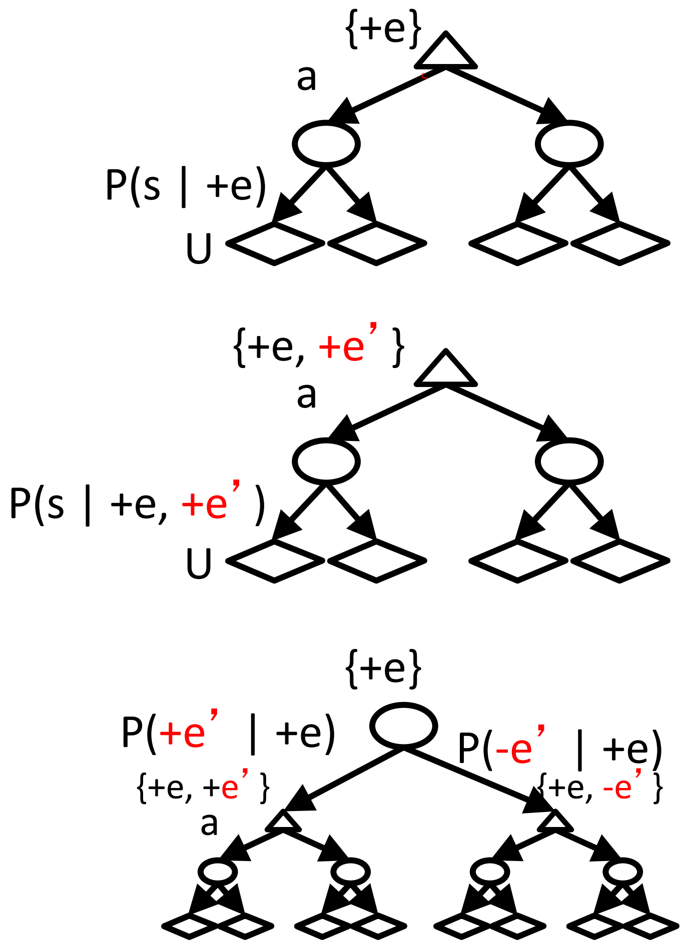
B)

- Expected value if E' is revealed and then we act:

$$\text{MEU}(e,E') = \sum_{e'} P(e'|e)\text{MEU}(e,e')$$

- Value of information: how much MEU goes up by revealing E' first then acting, over acting now:

$$\text{VPI}(E'|e) = \text{MEU}(e,E') - \text{MEU}(e)$$



VPI Properties

- Nonnegative

$$\forall E', e : \text{VPI}(E'|e) \geq 0$$

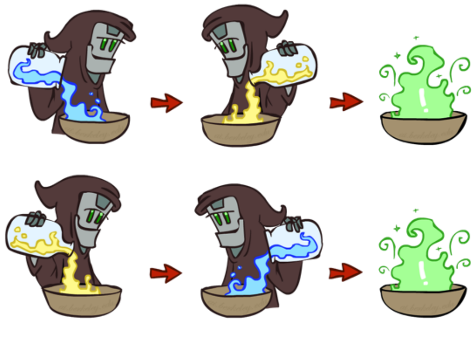
- Nonadditive

  (think of observing E_j twice)

$$\text{VPI}(E_j, E_k|e) \neq \text{VPI}(E_j|e) + \text{VPI}(E_k|e)$$

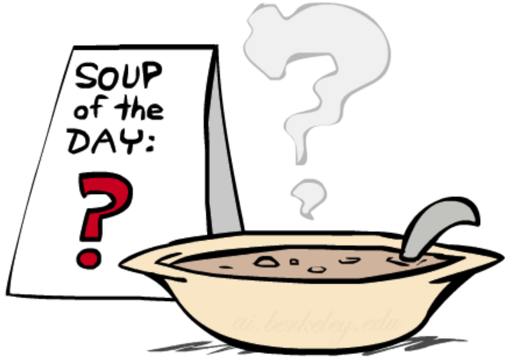
- Order-independent

$$\begin{aligned}\text{VPI}(E_j, E_k|e) &= \text{VPI}(E_j|e) + \text{VPI}(E_k|e, E_j) \\ &= \text{VPI}(E_k|e) + \text{VPI}(E_j|e, E_k)\end{aligned}$$



Quick VPI Questions

- The soup of the day is either clam chowder or split pea, but you wouldn't order either one. What's the value of knowing which it is?

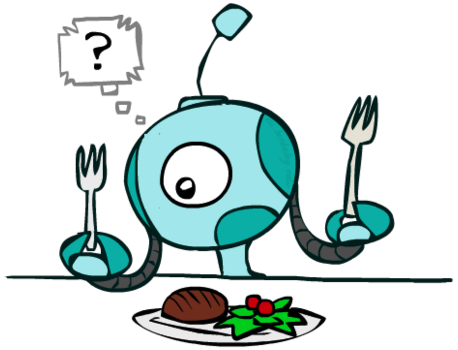


0

- There are two kinds of plastic forks at a picnic. One kind is slightly sturdier. What's the value of knowing which?

Exp Wings

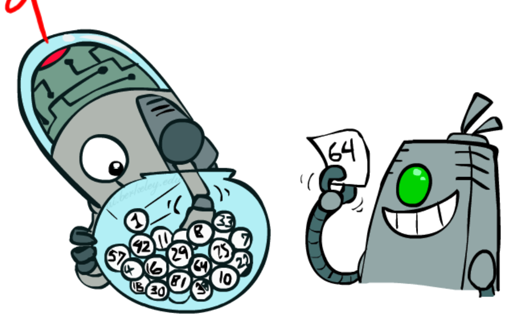
$\frac{1}{100} \cdot 100 = 1$



>0

- You're playing the lottery. The prize will be \$0 or \$100. You can play any number between 1 and 100 (chance of winning is 1%). What is the value of knowing the winning number?

$100 - 1 = 99$



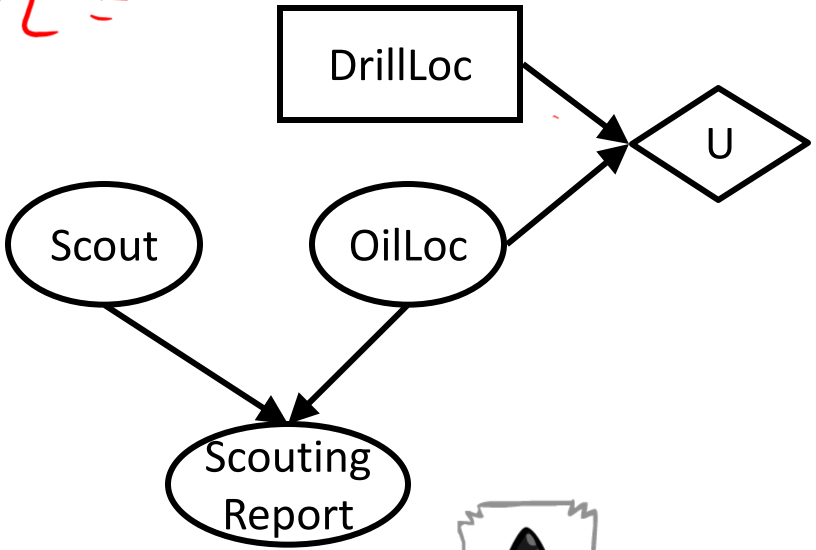
100-1
=99

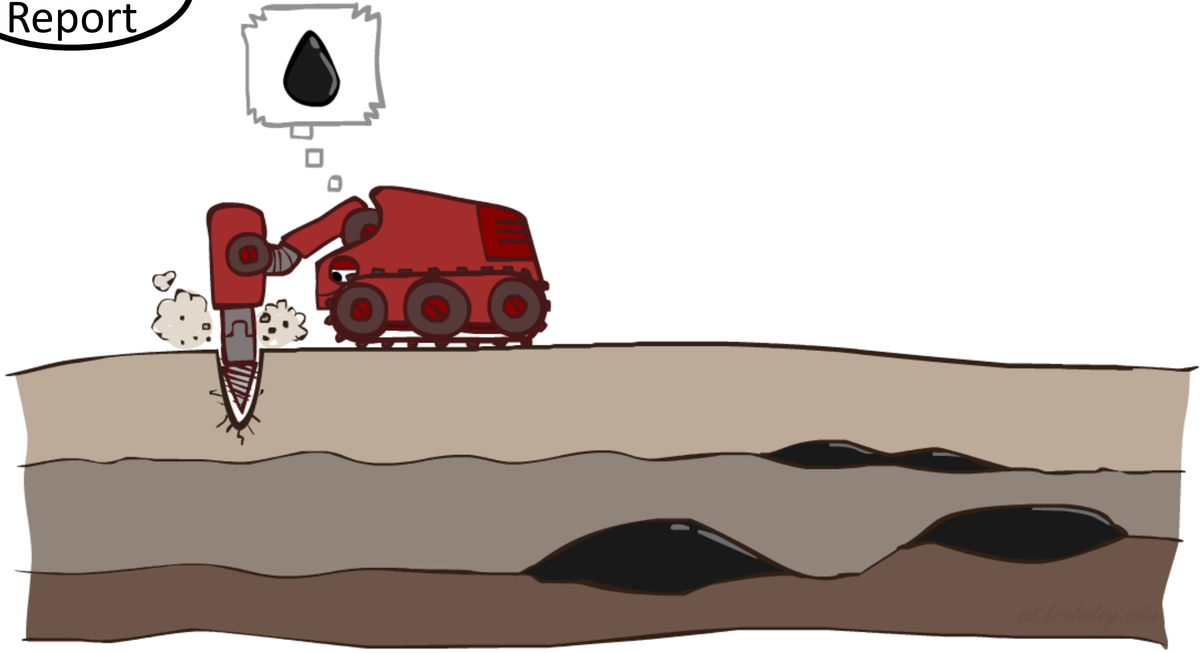
VPI Quiz

$K - K/2 =$

- VPI(OilLoc) ? k/2
- VPI(ScoutingReport) ? >0
- VPI(Scout) ? 0
- VPI(Scout | ScoutingReport) ?

  >0



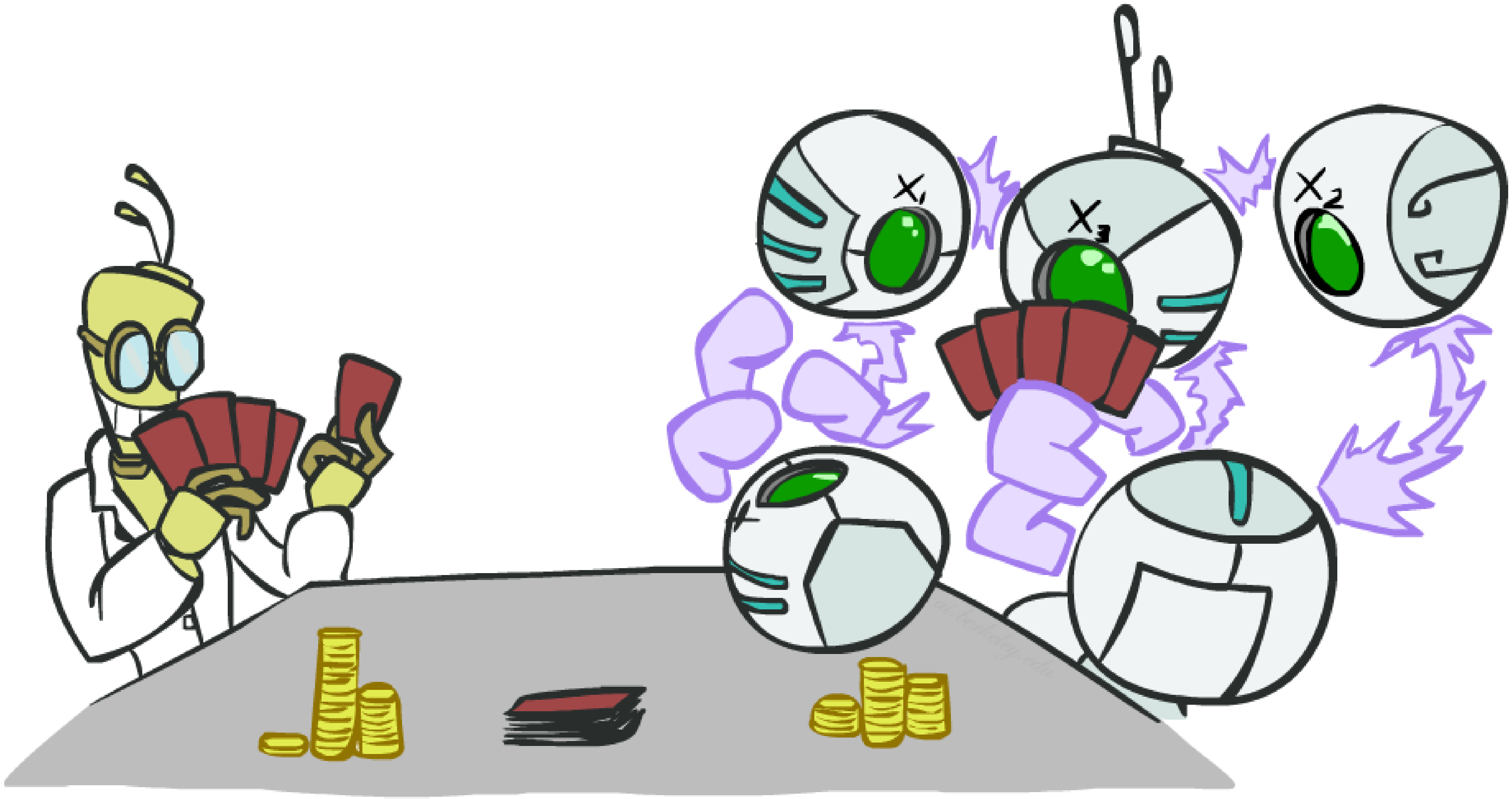


- Generally:

  If Parents(U) $\perp\!\!\!\perp$ Z | CurrentEvidence

  Then VPI(Z | CurrentEvidence) = 0

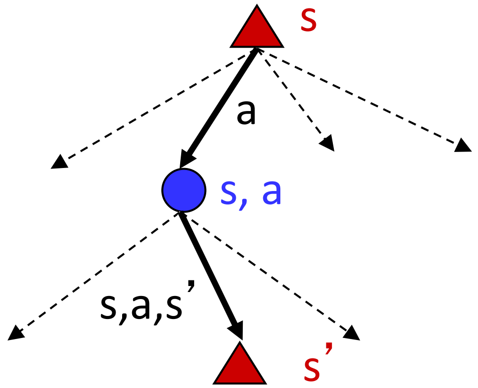
POMDP Teaser



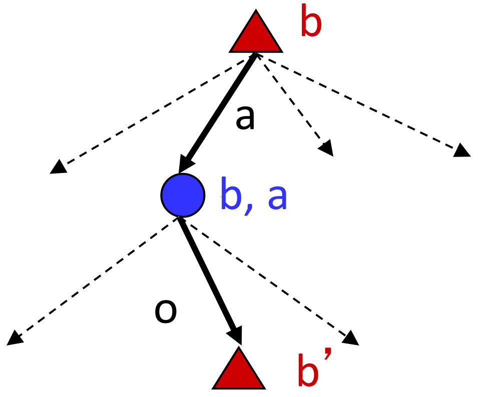
Partially Observable

POMDPs

- MDPs have: $= \{ s_1, s_2 \}$
  - States S
  - Actions A
  - Transition function P(s’ |s,a) (or T(s,a,s’))
  - Rewards R(s,a,s’)



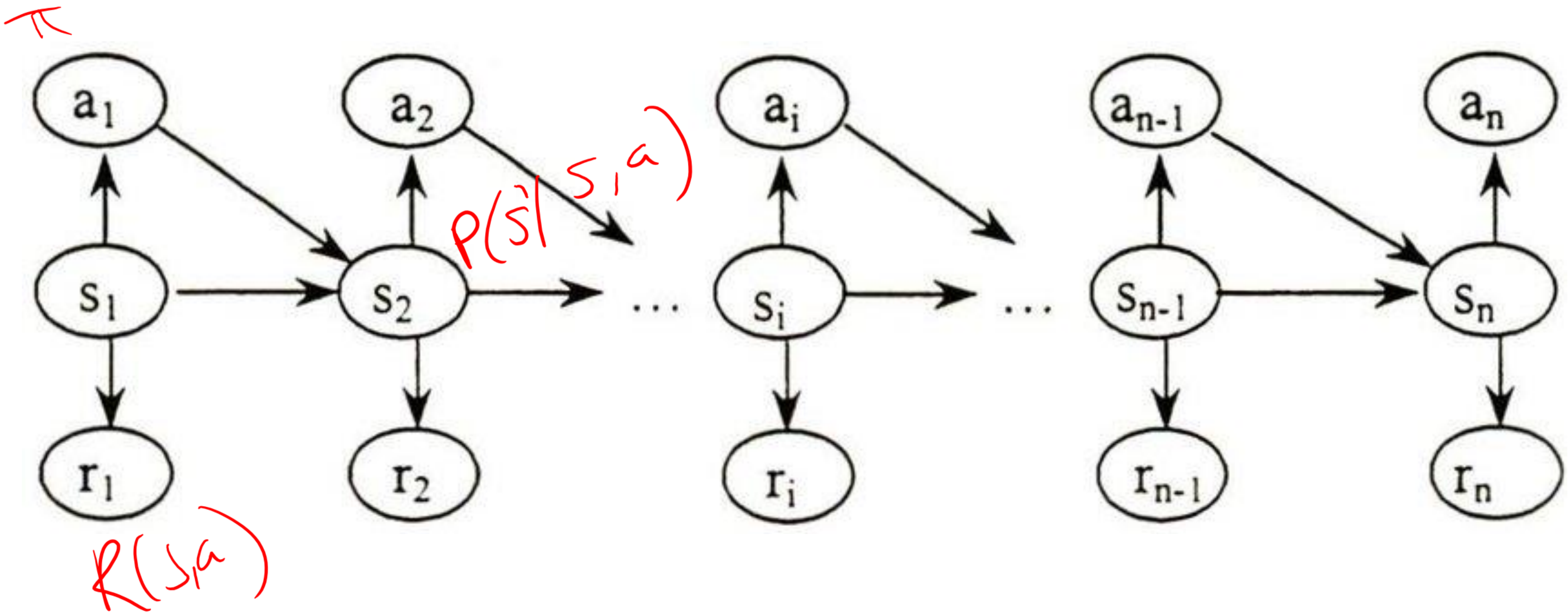
- POMDPs add:
  - Observations O
  - Observation function P(o|s) (or O(s,o))



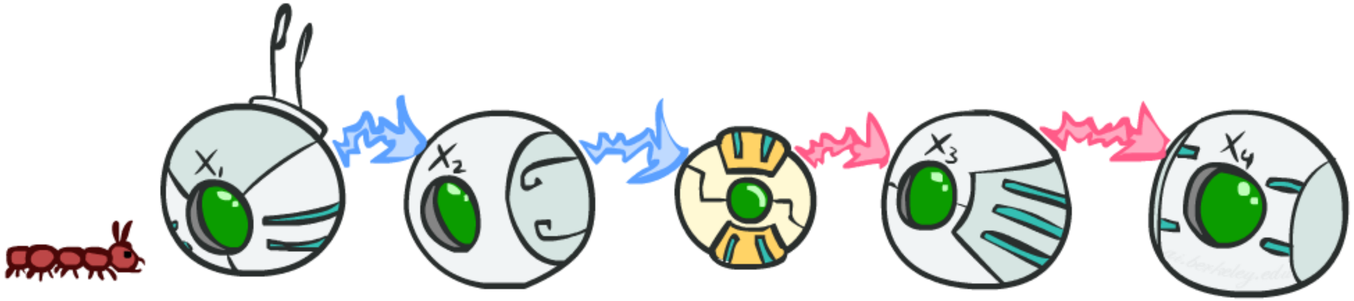
- POMDPs are MDPs over belief states b (distributions over S)

A popular and practical way to deal with partial observability is with an RNN!

How would we write an MDP as a Bayes' Net?



Markov Models: Bayes' Nets + Time

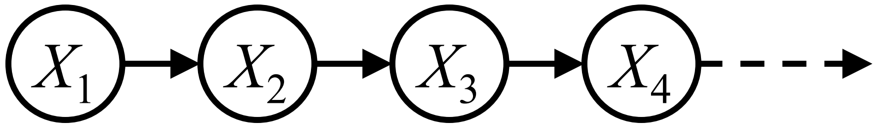


Reasoning over Time or Space

- Often, we want to reason about a sequence of observations
  - Speech recognition
  - Robot localization
  - User attention
  - Medical monitoring
- Need to introduce time (or space) into our models

Markov Models

- Value of X at a given time is called the state



$$P(X_1) \qquad P(X_t|X_{t-1})$$

- Parameters: called transition probabilities or dynamics, specify how the state evolves over time (also, initial state probabilities)
- Stationarity assumption: transition probabilities the same at all times
- Same as MDP transition model, but no choice of action

Joint Distribution of a Markov Model

$$X_1 \rightarrow X_2 \rightarrow X_3 \rightarrow X_4$$

$$P(X_1) \qquad P(X_t|X_{t-1})$$

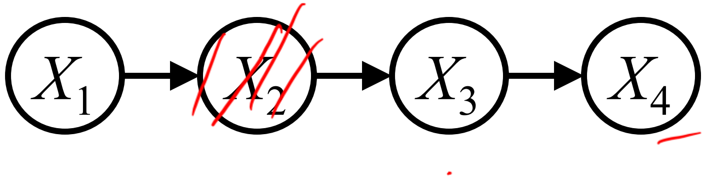
- Joint distribution:

$$P(X_1, X_2, X_3, X_4) = P(X_1)P(X_2|X_1)P(X_3|X_2)P(X_4|X_3)$$

- More generally:

$$\begin{aligned} P(X_1, X_2, \ldots, X_T) &= P(X_1)P(X_2|X_1)P(X_3|X_2)\ldots P(X_T|X_{T-1}) \\ &= P(X_1)\prod_{t=2}^{T} P(X_t|X_{t-1}) \end{aligned}$$

Implied Conditional Independencies



- Bayes' net implies $X_3 \perp\!\!\!\perp X_1 \mid X_2$ and $X_4 \perp\!\!\!\perp X_1, X_2 \mid X_3$

- Do we also have $X_1 \perp\!\!\!\perp X_3, X_4 \mid X_2$?
  - Yes!
  - D-Separation

Markov Models Recap

- Explicit assumption for all *t* : $X_t \perp\!\!\!\perp X_1, \ldots, X_{t-2} \mid X_{t-1}$
- Consequence, joint distribution can be written as:

$$P(X_1, X_2, \ldots, X_T) = P(X_1)P(X_2|X_1)P(X_3|X_2)\ldots P(X_T|X_{T-1})$$
$$= P(X_1)\prod_{t=2}^{T} P(X_t|X_{t-1})$$

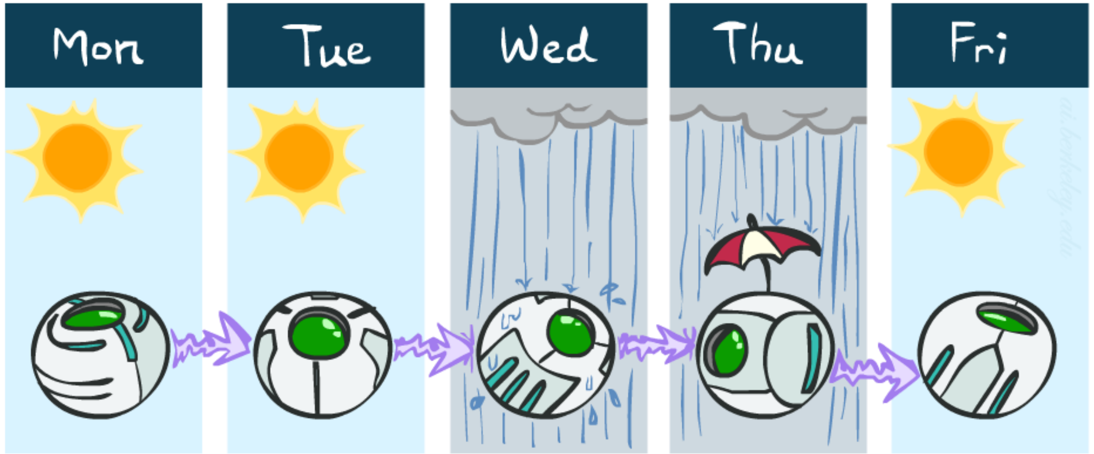
Huge savings in number of parameters needed!

- Implied conditional independencies:
  - Past variables independent of future variables given the present

  i.e., if $t_1 < t_2 < t_3$ or $t_1 > t_2 > t_3$ then: $X_{t_1} \perp\!\!\!\perp X_{t_3} \mid X_{t_2}$
- Additional explicit assumption: $P(X_t \mid X_{t-1})$ is the same for all *t*

Example Markov Chain: Weather

- States: $X = \{rain, sun\}$

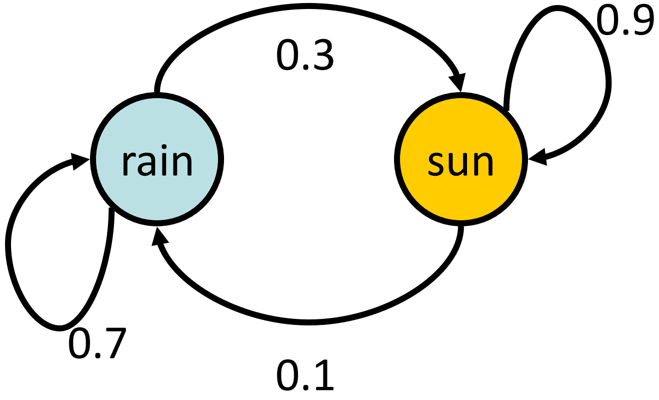


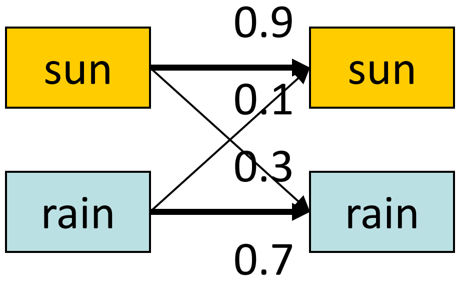
- Initial distribution: 1.0 sun

- CPT $P(X_t \mid X_{t-1})$:

X_{t-1}	X_t	$P(X_t \mid X_{t-1})$
sun	sun	0.9
sun	rain	0.1
rain	sun	0.3
rain	rain	0.7

Two new ways of representing the same CPT

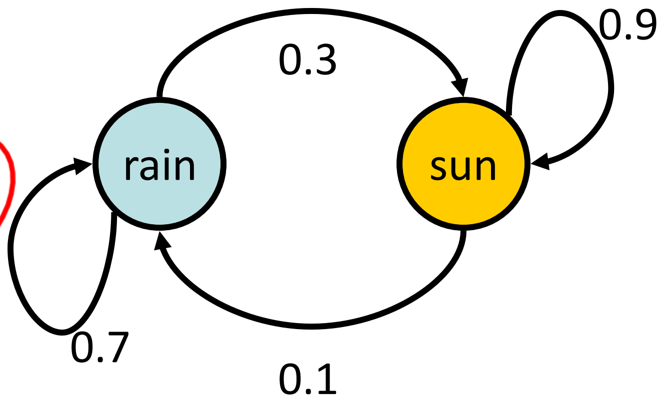




Example Markov Chain: Weather

- Initial distribution: 1.0 sun

$X_1 \to X_2 \to X_3$



- What is the probability distribution after one step?

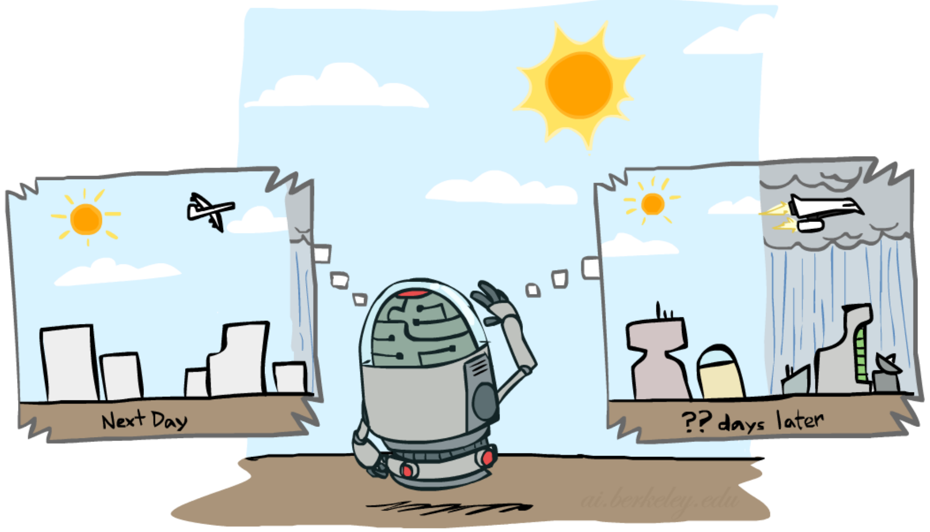
$$P(X_2 = \text{sun}) =$$

$$0.9 \cdot 1.0 + 0.3 \cdot 0.0 = 0.9$$

Mini-Forward Algorithm

- Question: What's P(X) on some day t?

$$X_1 \to X_2 \to X_3 \to X_4 \dashrightarrow$$

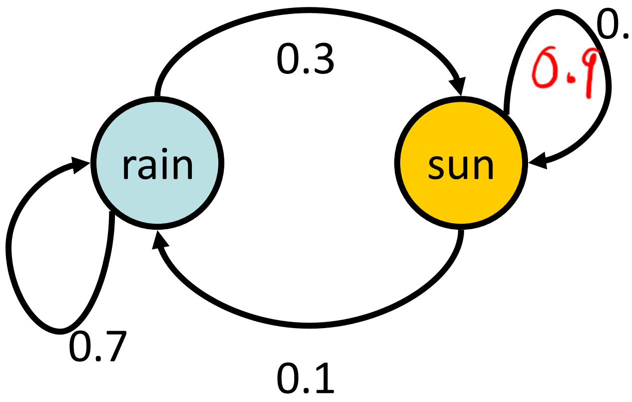


$$P(x_1) = \text{known}$$

$$P(x_t) = \sum_{x_{t-1}} P(x_{t-1}, x_t)$$
$$= \sum_{x_{t-1}} P(x_t \mid x_{t-1}) P(x_{t-1})$$

Forward simulation

Example Run of Mini-Forward Algorithm



- From initial observation of sun

s
r

$$\underset{P(X_1)}{\left\langle \begin{matrix} 1.0 \\ 0.0 \end{matrix} \right\rangle} \quad \underset{P(X_2)}{\left\langle \begin{matrix} 0.9 \\ 0.1 \end{matrix} \right\rangle} \quad \underset{P(X_3)}{\left\langle \begin{matrix} 0.84 \\ 0.16 \end{matrix} \right\rangle} \quad \underset{P(X_4)}{\left\langle \begin{matrix} 0.804 \\ 0.196 \end{matrix} \right\rangle} \Longrightarrow \underset{P(X_\infty)}{\left\langle \begin{matrix} 0.75 \\ 0.25 \end{matrix} \right\rangle}$$

- From initial observation of rain

$$\underset{P(X_1)}{\left\langle \begin{matrix} 0.0 \\ 1.0 \end{matrix} \right\rangle} \quad \underset{P(X_2)}{\left\langle \begin{matrix} 0.3 \\ 0.7 \end{matrix} \right\rangle} \quad \underset{P(X_3)}{\left\langle \begin{matrix} 0.48 \\ 0.52 \end{matrix} \right\rangle} \quad \underset{P(X_4)}{\left\langle \begin{matrix} 0.588 \\ 0.412 \end{matrix} \right\rangle} \Longrightarrow \underset{P(X_\infty)}{\left\langle \begin{matrix} 0.75 \\ 0.25 \end{matrix} \right\rangle}$$

- From yet another initial distribution $P(X_1)$:

$$\underset{P(X_1)}{\left\langle \begin{matrix} p \\ 1-p \end{matrix} \right\rangle} \quad \cdots \Longrightarrow \underset{P(X_\infty)}{\left\langle \begin{matrix} 0.75 \\ 0.25 \end{matrix} \right\rangle}$$

Stationary Distributions

- For most chains:
  - Influence of the initial distribution gets less and less over time.
  - The distribution we end up in is independent of the initial distribution

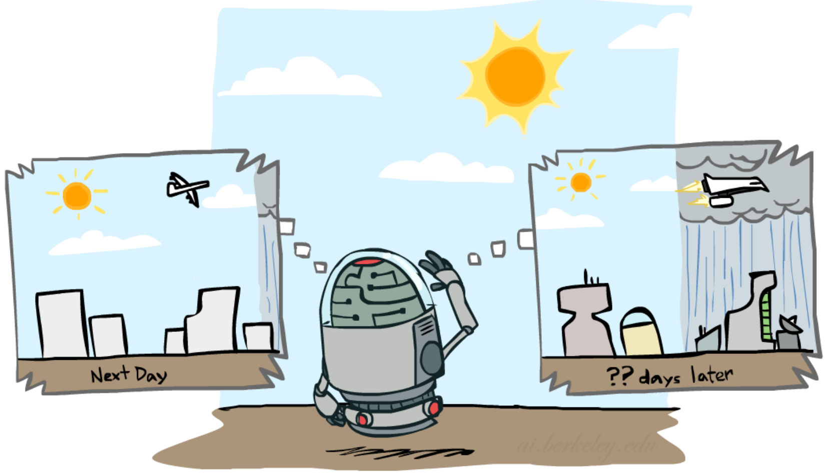
- Stationary distribution:
  - The distribution we end up with is called the stationary distribution P_∞ of the chain
  - It satisfies

$$P_\infty(X) = P_{\infty+1}(X) = \sum_x P(X|x)P_\infty(x)$$

Practice: Stationary Distributions

- Question: What's P(X) at time t = infinity?

$$X_1 \rightarrow X_2 \rightarrow X_3 \rightarrow X_4 \dashrightarrow$$



$$P_\infty(sun) = P(sun|sun)P_\infty(sun) + P(sun|rain)P_\infty(rain)$$
$$P_\infty(rain) = P(rain|sun)P_\infty(sun) + P(rain|rain)P_\infty(rain)$$

$$P_\infty(sun) = 0.9P_\infty(sun) + 0.3P_\infty(rain)$$
$$P_\infty(rain) = 0.1P_\infty(sun) + 0.7P_\infty(rain)$$

$$P_\infty(sun) = 3P_\infty(rain)$$
$$P_\infty(rain) = 1/3P_\infty(sun)$$

$$1 - P_\infty(r) = 3P_\infty(r)$$
$$1 = 4P_\infty(r)$$

Also: $P_\infty(sun) + P_\infty(rain) = 1$

$$\Rightarrow \quad P_\infty(sun) = 3/4 \qquad P_\infty(rain) = 1/4$$

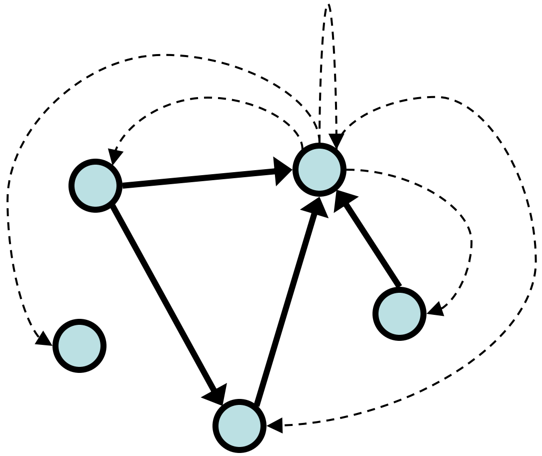
X_{t-1}	X_t	$P(X_t \mid X_{t-1})$
sun	sun	0.9
sun	rain	0.1
rain	sun	0.3
rain	rain	0.7

Is this useful? Can this make you money?

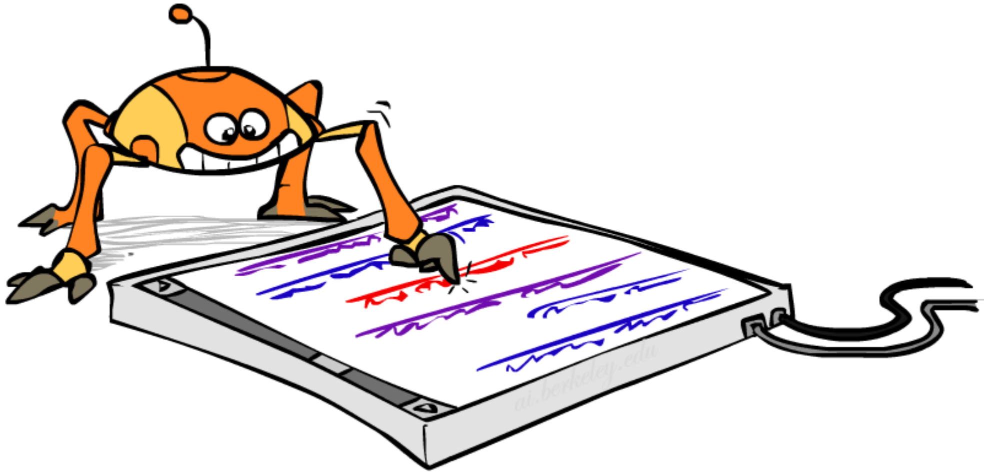

Google
Google
Google

Application of Stationary Distribution: Web Link Analysis

- PageRank over a web graph
  - Each web page is a state
  - Initial distribution: uniform over pages
  - Transitions:
    - With prob. c, uniform jump to a random page (dotted lines, not all shown)
    - With prob. 1-c, follow a random outlink (solid lines)



- Stationary distribution
  - Will spend more time on highly reachable pages
  - E.g. many ways to get to the Acrobat Reader download page
  - Somewhat robust to link spam
  - Google 1.0 returned the set of pages containing all your keywords in decreasing rank, now all search engines use link analysis along with many other factors (rank actually getting less important over time).
  - User behavior is now very important (what people click on)

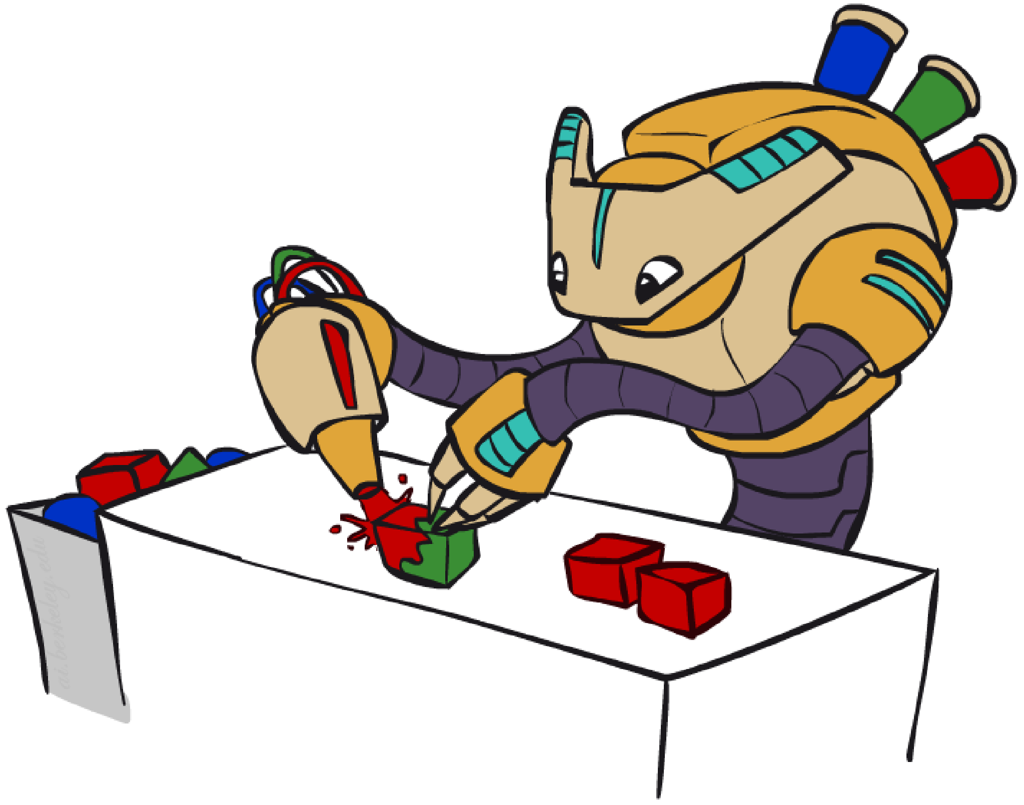


Application of Stationary Distributions: Gibbs Sampling

- Each joint instantiation over all hidden and query variables is a state: $\{X_1, \ldots, X_n\} = H \cup Q$
- Transitions:
  - With probability 1/n resample variable X_j according to

    $$P(X_j \mid x_1, x_2, \ldots, x_{j-1}, x_{j+1}, \ldots, x_n, e_1, \ldots, e_m)$$

- Stationary distribution:
  - Conditional distribution $P(X_1, X_2, \ldots, X_n \mid e_1, \ldots, e_m)$
  - Means that when running Gibbs sampling long enough we get a sample from the desired distribution
  - Requires some proof to show this is true!



Next Time: Hidden Markov Models!